Draft Maps for Arizona's 2021 Congressional Districts

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This research analysis was conducted as part of the Institute for Computational Redistricting (ICOR) at the University of Illinois at Urbana-Champaign. This activity was conducted in a non-partisan manner, with any political descriptors used reflecting the results of the quantitative analysis, not the opinions of the researchers nor ICOR.

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Executive Summary

The Institute for Computational Redistricting (http://redistricting.cs.illinois.edu) is a research group at the University of Illinois at Urbana-Champaign. Under the advisement of Dr. Sheldon H. Jacobson (http://shj.cs.illinois.edu) and Dr. Douglas M. King, the group focuses on computational methods for redistricting, and provides transparency within the redistricting process. Following the 2020 census, the Arizona Independent Redistricting Commission (IRC) seeks draft maps for state and congressional district maps. In this report, we present congressional district maps for Arizona created using optimization-based computational tools. The goals of this endeavor are to promote the use of quantitative methodologies for transparent redistricting and in particular to examine the implications for redistricting in Arizona.

Proposition 106 of the Arizona Constitution lists six key criteria to be incorporated in the creation of district maps. These include creating contiguous districts with roughly equal population and compact shapes, incorporating federal Voting Rights Act requirements to create majority-minority districts, preserving communities of interest and geographical features such as counties and census tracts, and favoring competitive districts. In the language used in the Arizona Constitution, the majority of the criteria are expected to be satisfied "to the extent practicable". However, improving a district plan's performance under one criterion often requires some compromise in other criteria. For example, if census tracts cannot be divided into census blocks, it is not possible to achieve a "1-person" population deviation. In this report, we use computational tools to explore and analyze the trade-offs between population balance, competitiveness and the compactness of districts.

We provide six congressional district maps for Arizona created using an optimization algorithm. To create these maps, the geographic and demographic data are obtained from the U.S. Census Bureau (2020), and the voting data is aggregated from nine elections in 2016 and 2018 obtained from OpenPrecincts (2020). Using this data, these six maps have been optimized to find compact and/or competitive districts while ensuring that other criteria in Proposition 106 (including population balance, contiguity, unbroken census tracts, requisite number of majority-minority districts) are satisfied relative to existing norms. The maps are labeled A through F, and are presented in detail in Section 4 of this report. Maps A, B, D and E are optimized for competitiveness while being reasonably compact. Maps C and F are optimized for compactness. In each map, the district populations are allowed to deviate from the ideal district population by a specified tolerance percentage. The deviation tolerance for maps A through C is set to 0.5%, while the tolerance for maps D through F is set to 0.1%. All of these maps are created with census tracts as the basic building blocks while keeping 9 of the 15 counties intact. Each map has at least 3 majority-minority districts to satisfy the Voting Rights Act requirements.

Competition can be measured either by the maximum vote spread (i.e., the maximum percentage difference in the Democratic and Republican voters among all the districts), or by the number of *competitive* districts in that map. Here, a district is *competitive* if its vote spread is within 7%. In the most competitive maps A and D obtained by the optimization algorithm, all the district vote spreads are within 7% and 7.46%, respectively. In contrast, the 117th Congressional district map has a maximum vote spread of 36.45%. In the district maps that emphasize competitiveness, it is observed that each district evenly divides the Republicans and Democrats voters. In particular, the districts that are entirely contained within Maricopa County evenly divide the Democratic voters in urban Phoenix area and the Republican voters in the areas surrounding Phoenix.

Compactness can be measured by the total perimeter length (in km) of all the districts; a smaller perimeter value is preferred. Maps C and F are the most compact maps obtained by the optimization algorithm with perimeter values of 3,933 km and 4,575 km, respectively. In contrast, the 117th Congressional district map has a perimeter of 6,614 km. To emphasize compactness, several of the districts in maps C and F are safe districts (i.e., with vote spread more than 7%) for either of the two parties. In particular, the spatial distribution of the two parties' voters invariably gives Republicans an advantage over Democrats in terms of the number of safe districts created when compactness is emphasized.

The maps presented in this report highlight the ability of the optimization algorithm to find district maps that emphasize particular redistricting criteria such as competitiveness or compactness. If other criteria should be emphasized instead of competitiveness and compactness, the algorithm is flexible enough to accommodate those as well. For example, if it is desired to create district maps that have a "1-person" population deviation, certain census tracts can be broken into census blocks to achieve highly equipopulous district maps. Further, the algorithm is also flexible enough to accommodate other criteria not considered in the presented maps, including the preservation of communities of interest. Beyond the creation of these maps, this endeavor illustrates the positive impact from adapting computationally transparent methodologies for political redistricting.

1. Introduction

In November 2000, Arizona amended their state constitution following a citizen initiative to assign the power to draw congressional and state legislative districts to a newly created Independent Redistricting Commission (IRC). Since then, the IRC has been tasked with redrawing Arizona's district maps following every decennial census. This amendment, called *Proposition 106*, outlines specific redistricting criteria such as creating contiguous districts with roughly equal population and compact shapes, incorporating federal Voting Rights Act requirements to create majority-minority districts, preserving communities of interest and geographical features such as counties and census tracts, and favoring competitive districts. Since the 2020 census data has been released, the IRC has invited proposals for draft district maps.

During the last decade, there have been significant advances in computational tools to draw district maps grounded in fundamental research spanning fields such as computer science, operations research, mathematics, geography and political science. Our research group, the Institute of Computational Redistricting (ICOR), specializes in optimization algorithms for political redistricting with the goal of creating a transparent process for computational redistricting. An optimization algorithm is one which computationally seeks the best solution (i.e., a district map) within the scope of the legislative criteria provided to the algorithm. This method is well suited to fit the needs of a redistricting process with clearly-defined requirements such as Arizona's redistricting process.

This report provides six draft maps for Arizona's nine congressional districts. The optimization algorithm used to create these maps is tailored to the six criteria in Arizona's Proposition 106. In particular, these maps have been optimized to find the most compact and/or competitive districts while ensuring that other criteria (including population balance, contiguity, requisite number of majority-minority districts) are satisfied relative to existing norms. Each of the six maps emphasizes certain criteria more than others, and the maps highlight the inherent trade-offs among the criteria. For example, a highly competitive map may not be as compact. The maps also underscore the ability of the optimization algorithm to find maps that perform well for any specified objective; the optimization algorithm can be adapted to serve any additional/alternative criteria not considered in this report. For example, once the communities of interest have been finalized by the IRC, the algorithm can create districts that preserve these communities.

This report is organized as follows. In Section 2, we discuss background information on the redistricting requirements in Arizona, existing metrics for partisan fairness in redistricting, and the datasets used in this report. In Section 3, we outline the optimization algorithm used to create the district maps presented in this report. In Section 4, we present six district maps, where each map emphasizes a different degree of trade-off between the redistricting requirements from the Arizona State Constitution. In Section 5, we discuss these maps in detail and summarize key takeaways for the IRC and other Arizona stakeholders.

2. Redistricting in Arizona

This section describes how congressional redistricting is conducted in Arizona. We discuss the redistricting requirements in the Arizona Constitution, partisan fairness metrics from political sciences that model aspects of these requirements, and the data sources used by our algorithm to create the district plans.

2.1. Constitutional Requirements

This section excerpts relevant text from the Arizona State Constitution describing the redistricting requirements (Arizona, 2021) and discusses how each of them is addressed in this report. The maps in this report satisfy these requirements to the extent possible. Arizona's redistricting requirements are:

A. "Districts shall comply with the United States Constitution and the United States voting rights act;"

The landmark U.S. Supreme Court cases of the 1960s, including *Baker v. Carr* (1962), *Wesberry v. Sanders* (1964), and *Reynolds v. Sims* (1964), established the "one person, one vote" principle, meaning congressional districts within a state should have approximately the same populations to provide equal representation to all constituents (McGhee, 2020). We discuss population balance in more detail under requirement B.

The Voting Rights Act of 1965, which prohibits racial discrimination in voting, has been interpreted by the U.S. Supreme Court in *Thornburg v. Gingles* (1986) to support the creation of majority-minority districts when justified by a large enough concentration of a minority group (Niemi, Grofman, Carlucci, & Hofeller, 1990). A district is a majority-minority district if a minority group or a collection of minority groups comprise a simple majority in the district's population (Ballotpedia, 2021). This report creates majority-minority districts for each draft map.

B. "Congressional districts shall have equal population to the extent practicable, and state legislative districts shall have equal population to the extent practicable;"

In pursuit of population balance, we constrain all district populations to be within ± 0.5% of the ideal (average) population. Achieving near-perfect population balance in an optimization framework is generally considered impractical (Ricca, Scozzari, & Simeone, 2013; Validi, Buchanan, & Lykhovyd, 2021), so a 1% range between the largest and smallest populations is often used when producing draft maps (Altman & McDonald, 2018). As noted by an expert report for the 2018 redistricting litigation in Pennsylvania, an algorithmically produced map with such a small population deviation may be manually balanced to within one person by a human mapmaker via small adjustments (Duchin, 2018). Other redistricting objectives, such as preservation of political boundaries or communities of interest, may justify small population deviations; the current West Virginia districts are able to preserve all counties by allowing a 0.79% population deviation (NCSL, 2020).

C. *"Districts shall be geographically compact and contiguous to the extent practicable;"*

Intuitively, a district is compact if its boundary resembles a simple shape, such as a circle, rectangle, or regular polygon. But mathematical definitions of compactness abound (Grofman & Cervas, 2021), and survey results suggest none of the existing definitions fully align with human perception of compactness (Kaufman, King, & Komisarchik, 2021). Despite the lack of a standard definition, compactness is considered by judges and political scientists to be an important guard against gerrymandering (Kaufman, King, & Komisarchik, 2021). A noncompact district (i.e., one with a convoluted shape) not only suggests nefarious intent but also makes representation more confusing for both the representative and their constituents (Grofman & Cervas, 2021).

For this report, we measure compactness as the sum of all district perimeters (reported in km). The reported sum excludes perimeter segments that coincide with Arizona's state boundary and are therefore district boundaries in any map. Other things being equal, a district with an irregular shape has a larger perimeter than a district with a simpler shape, as shown in Figure 1. Hence smaller values of the sum-of-perimeters metric indicate more compact districts.

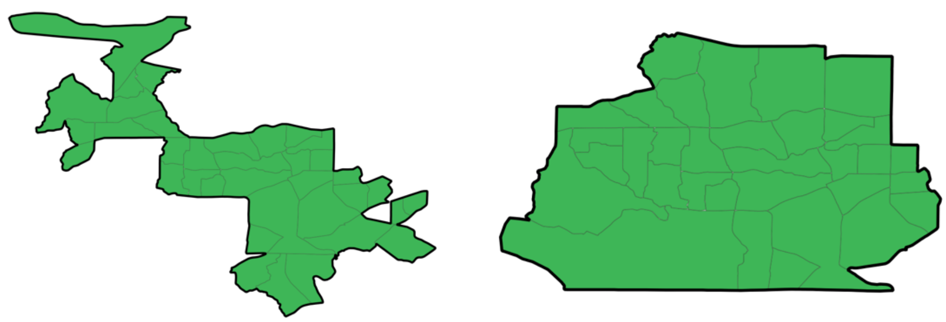


Figure 1. Examples of noncompact (left) and compact (right) districts. The perimeter of the left district is 317 km, whereas the perimeter of the right district is 151 km.

We enforce contiguity using adjacency data, where two census tracts are considered adjacent if they share a border of nonzero length. Using these adjacencies, the optimization algorithm computationally requires that one must be able to travel from any census tract in a district to any other census tract in the district by passing through a sequence of adjacent census tracts within the district. Every census tract in Arizona is itself contiguous, so this guarantees contiguous districts.

D. *"District boundaries shall respect communities of interest to the extent practicable;"*

The IRC is in the process of collecting communities of interest from the public. Even though the draft maps in this report do not yet preserve communities of interest, this criterion can be added once the communities are finalized by the IRC. Further, as a byproduct of keeping census tracts intact (see the below discussion of requirement E), the maps presented in this report likely respect most communities of interest with small land area. Larger communities of interest benefit from the effort to split as few counties as possible.

E. *"To the extent practicable, district lines shall use visible geographic features, city, town and county boundaries, and undivided census tracts;"*

The maps produced in the report use undivided census tracts as the basic building blocks. Furthermore, we only split the six most populous counties, so many county boundaries are preserved, along with municipal boundaries that do not cross census tracts. Additionally, similar to communities of interest, other visible geographic features can also be preserved by the algorithm if the process requires it.

F. *"To the extent practicable, competitive districts should be favored where to do so would create no significant detriment to the other goals."*

Maintaining competitive districts can encourage voter turnout, reduce district packing, and discourage candidate complacency (Hirano & Snyder Jr., 2012; McCarty, Poole, & Rosenthal, 2009; Tapp, 2019). As with compactness, competitiveness admits a plethora of candidate definitions (DeFord, Duchin, & Solomon, 2020). As a measure of competitiveness, the IRC reports each district's *vote spread*, the difference between the average Democratic and average Republican votes in recent elections, as a measure of competitiveness, describing districts with a vote spread less than 4% as "highly competitive" and between 4% and 7% as "competitive" (Arizona IRC, 2021). Each draft map in this report includes the vote spread for each district and identifies competitive and highly competitive districts.

2.2. Partisan Fairness Measures

Political scientists and redistricting reformers often care about partisan fairness measures beyond competitiveness. While these other partisan fairness measures do not appear in Arizona's constitutional requirements, we report them to provide a more comprehensive picture of the trade-offs for each draft map.

The first partisan fairness measure reported is the *efficiency gap*, the difference between the two parties' wasted votes divided by the total number of votes; votes are considered wasted if they are for the party expected to lose a district or are in excess of the 50% needed to win a district (Stephanopoulos & McGhee, 2015). The efficiency gap of a district map quantifies imbalances in how many of each party's voters are *packed* into districts where their candidate wins by a landslide and/or *cracked* across districts where their candidate loses narrowly. Based on a historical analysis of the efficiency gap in U.S. district maps, its creators suggest a ± 8% threshold as reasonable in terms of partisan fairness (Stephanopoulos & McGhee, 2015). While academics debate the merits of the efficiency gap (Chambers, Miller, & Sobel, 2017; Cho, 2017; Stephanopoulos & McGhee, 2018), it has been used in in the courts to help overturn gerrymandered maps (Bernstein & Duchin, 2017), so we include it in this report.

We also include the *partisan symmetry* of each draft map in this report. Partisan symmetry compares party seat-shares under hypothetical shifts in the vote-share across all districts (Grofman & King, 2007). Such counterfactual analysis produces a *seats-votes* curve for each party, with vote-share on the x-axis and seat-share on the y-axis. The asymmetry of the map is then calculated as the area between the two parties' seats-votes curves (Grofman, 1983; Katz, King, & Rosenblatt, 2020). Though fraught with computational issues and paradoxes like the efficiency gap (DeFord, et al., 2021), partisan symmetry has earned moderate endorsements from judges (see *LULAC v. Perry* (2006)). We therefore include the partisan asymmetry score of each draft map in this report.

2.3. Data Sources

Redistricting in Arizona requires data describing the state's geography, population, racial/ethnic demographics, and election results. The data sources used to create the draft maps in this report are listed below.

- *State geography*: Districts comprise geographic units, such as census blocks, census block groups, census tracts, and counties. The 2020 U.S. Census provides spatial data for these units as shapefiles (U.S. Census Bureau, 2020). With spatial data, we can determine which units are adjacent (to enforce district contiguity) and the length of shared borders between adjacent units (to calculate district perimeters as a measure of compactness). We process the adjacency information from the shapefiles using a combination of the free and open-source QGIS software and the publicly available Shapely Python package.

  In the 2020 U.S. Census, Arizona's 15 counties contain 1,765 census tracts and 241,666 census blocks. To create the district maps presented in this report, we treat census tracts as indivisible units. Figure 2 indicates the six most populated counties of Arizona – Maricopa, Mohave, Pima, Pinal, Yavapai, and Yuma – that are split into census tracts in the creation of the district plans in this report. Figure 3 depicts all the census tracts and counties in AZ.

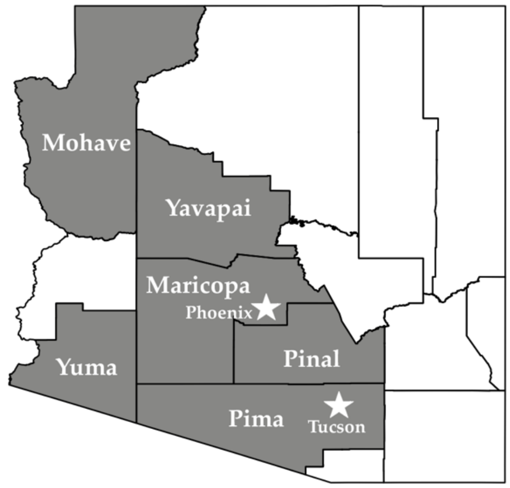


Figure 2. The six most populous counties in Arizona from the 2020 census.

- <u>*Population and racial demographics*</u>: In addition to spatial data, the U.S. Census also provides population counts and racial/ethnic demographics for all geographical units (U.S. Census Bureau, 2020). Our algorithm uses population data to ensure that all districts have approximately equal populations as discussed in Section 2.1. Figure 4 depicts the distribution of the population in Arizona. Note that in Figures 4-6, the census tracts in white contain zero population. Dividing Arizona's total population of 7,151,502 by nine districts gives an ideal congressional district population of 794,611.

  The racial/ethnic demographic data is necessary to require that a certain number of districts have a majority of the population consisting of minority groups as required by the Voting Rights Act. Figure 5 depicts the census tracts with the Non-Hispanic (N.H.) White majority populations and those with majority-minority populations. Overall, the N.H. White population comprise 53.4% of the population, while the 46.6% minority population can be broken down as follows: 30.7% Hispanic or Latino, 4.4% N.H. Black or African American, 3.7% N.H. American Indian and Alaska Native, 3.7% Asian, Native Hawaiian and Pacific Islander, and 4.1% N.H. other or multiple races.

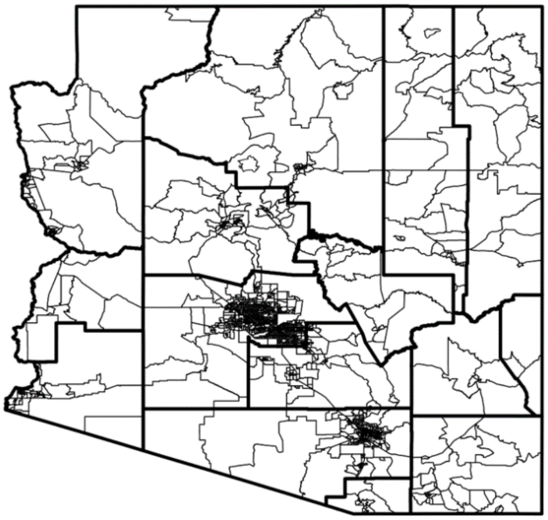


Figure 3. The 1,765 census tracts (thin lines) and 15 counties (thick lines) of Arizona.

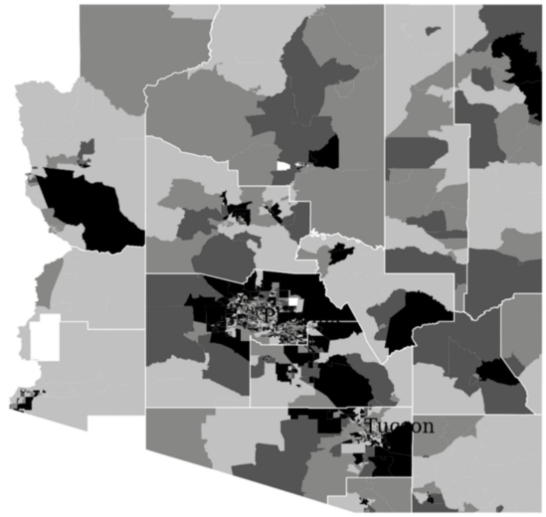


Figure 4. Arizona census tract populations, as of 2020. More populous census tracts are shaded darker than less populous tracts.

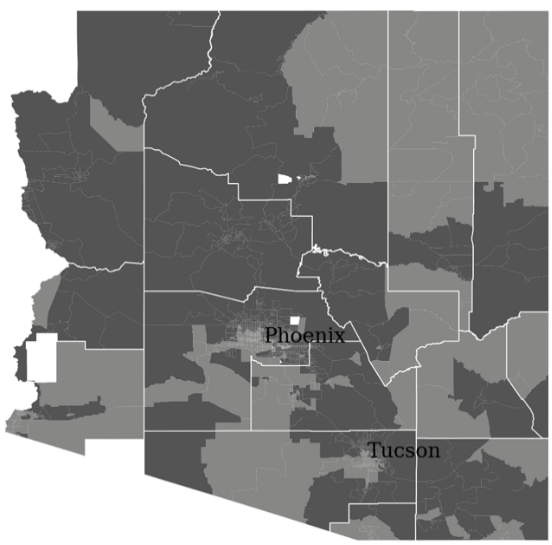


Figure 5. The racial/ethnic majorities in census tracts, as of 2020. The census tracts with majority Non-Hispanic White populations are in dark gray and those with majority-minority populations are in light gray.

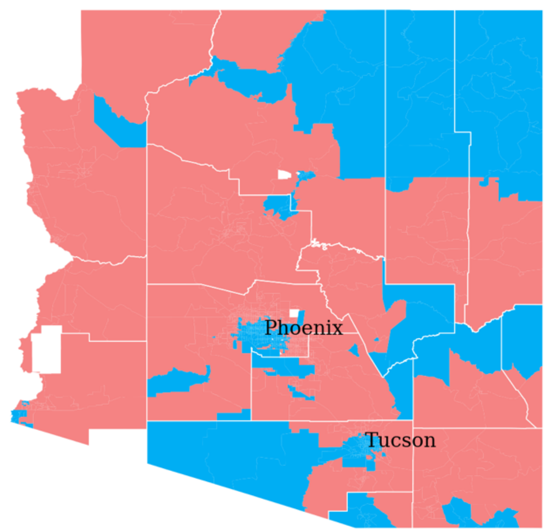


Figure 6. The party affiliation in each census tract obtained from aggregating nine specific election results in 2016 and 2018. Census tracts with Democratic majorities are in blue and those with Republican majorities are in red.

- *Election results*: Voting data from past elections are used to measure political fairness metrics, such as competitiveness, as discussed in Section 2.1. We aggregate the results from nine elections: 2016 President and U.S. Senate; 2018 U.S. Senate, Governor, Secretary of State, Attorney General, State Treasurer, Superintendent of Public Instruction and State Mine Inspector. Precinct-level voting data for these elections are available from OpenPrecincts (2020). Note that we only use voting data for the two major political parties.

  Since we create district maps using census tracts, the voting data needs to be transformed from the voting precincts to census tracts. Since census blocks are nested inside voting precincts, we disaggregate the precinct-level data to census blocks by distributing the voters in each precinct proportional to the populations of the census blocks in the precinct. Then, the voting data is aggregated to the census tracts since census blocks nest inside census tracts.

  With this voting data, we can examine Arizona's political geography. Based on the nine focus elections, Arizona voters are roughly 48.3% Democrat and 51.7% Republican. Figure 6 illustrates the distribution of Democratic and Republican voters in the state.

3. Optimization Algorithm

This section describes the optimization algorithm used to create district maps. This algorithm operates in two stages. In Stage 1, an initial district map is created using the initial legal criteria provided to the algorithm. In Stage 2, this district map is adjusted to improve an objective, such as compactness or competitiveness, while continuing to satisfy the legal criteria. Figure 7 provides an overview of the optimization framework as a flowchart.

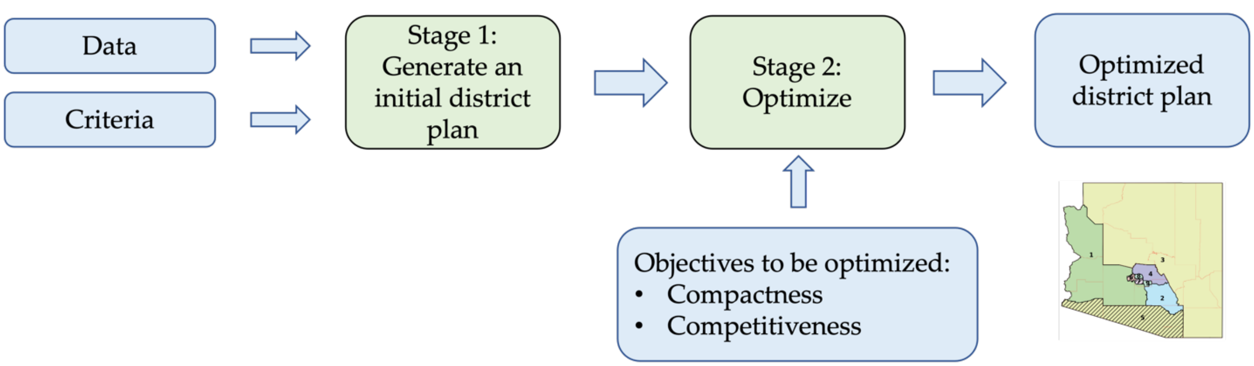


Figure 7. An overview of the optimization framework.

3.1. Generating an initial district map

The goal of Stage 1 of the algorithm is to create an initial district map that satisfies all the legal criteria that are provided to the algorithm. The geographical, demographic, and voting data gathered for the census tracts are used by the algorithm to ensure that the legal criteria are satisfied. We use the following legal criteria to create the initial district maps presented in this report.

a. All the districts must be **contiguous.**
b. The **population** in each district must not deviate from the ideal district population by more than a pre-specified tolerance. We use two deviation tolerances, namely ± 0.5% and ± 0.1%. As noted in Section 2.1, a deviation of up to 0.5% is often used when producing draft maps (Altman & McDonald, 2018), which can be manually balanced to within one person by a human mapmaker via small adjustments (Duchin, 2018; Swamy, King, & Jacobson, 2019).
c. The number of **counties to be split** into census tracts cannot be more than a pre-specified value. We set the 6 most-populous counties be split into census tracts. In contrast, the current congressional district map for the 117th Congress (2021-2023) splits 7 counties.
d. The number of **majority-minority districts** must be at least a pre-specified value. We require a minimum of 3 majority-minority districts, which is the same as in the current congressional district map.
e. The number of **census tracts split** into census blocks must not be more than a pre-specified value. In our district maps, this value is set to zero. Even though we do not split any census tract into census blocks, this functionality exists to satisfy smaller population deviations. In contrast, the current congressional district map splits 151 census tracts into census blocks.

Other criteria can also be included in this framework. For example, once the communities of interest are finalized by the IRC, the algorithm can create district maps that keep each community intact within a district.

To create an initial district map in Stage 1, we apply a combination of computational tools and optimization approaches from Swamy et al. (2019), Bozkaya et al. (2003) and DeFord et al. (2019). The input to Stage 1 is the set of geographical units comprising the 9 counties that are preserved and the census tracts in the 6 counties that are split. There are four phases to creating an initial district map in Stage 1.

i. *Coarsen*: In the first phase, select neighboring geographical units are merged to create a smaller set of units using a procedure called *coarsening* as proposed in Swamy et al. (2019). The advantage of coarsening is that the algorithm is computationally faster when creating districts using a smaller set of units as opposed to doing so with a large number of census tracts/blocks.
ii. *Create a contiguous and equipopulous map*: In the second phase, using the smaller set of geographical units, a contiguous and equipopulous (within the given tolerance) district map is drawn using a *local search* method proposed by Bozkaya et al. (2003). In this method, the units are first randomly grouped together into contiguous districts without requiring the districts to be equipopulous, and the population imbalance is

improved in several steps. Each step transfers a randomly chosen unit from one district to another. These steps continue until the district populations cannot be further improved. At the end of this procedure, if the district populations are within the population deviation tolerance, then the resulting contiguous and equipopulous district map moves on to Phase iii; otherwise, Phase ii is repeated from scratch until the populations are within the population deviation tolerance. The smaller the population deviation tolerance, the more iterations may be needed before an equipopulous district is found.

iii. <u>*Create majority-minority districts:*</u> In the third phase, a requisite number of majority-minority districts are created by using a second local search method from DeFord et al. (2019). Like the local search method proposed by Bozkaya et al. (2003), this method also improves a given district map by making small changes in several steps. However, instead of moving one unit at a time, this method merges two neighboring districts and completely redraws the boundary between them. This was found to be a faster improvement method provided that the starting district map was already equipopulous. Using this method, the district map from the second phase is adjusted until the number of majority-minority districts is at least the requisite number specified in the criteria.

iv. <u>*Improve compactness:*</u> Even though the district map from the third phase satisfies the legal criteria provided, the districts are often noncompact. The map's compactness is improved by the local search method of DeFord et al. (2019) running for 1,000 steps.

v. <u>*Uncoarsening:*</u> The resulting map from Phase iv is transformed back to the original geographical units (preserved counties, and census tracts of broken counties) using a procedure called *un-coarsening* (Swamy, King, & Jacobson, 2019). The resulting map satisfies the legal criteria provided to the algorithm.

3.2. Optimizing a district map

The goal of Stage 2 of the optimization algorithm is to find the *best* (or *optimal*) district map possible within the legal constraints provided to the algorithm. Here, the quality of a map is assessed by an *objective* of choice such as compactness or competitiveness. Due to the computationally challenging nature of optimizing a district map, an optimization algorithm may not be able to identify an optimal district plan within a practical span of time. Hence, the goal of Stage 2 of our optimization algorithm is to find a district map that optimizes the specified objective to the extent possible under these computational limitations.

While an optimization algorithm can draw a map from scratch, providing the algorithm with an initial map that already satisfies the legal criteria can help the optimization algorithm find good maps more quickly, thereby easing some of its computational burden. Hence, we use the initial district map constructed in Stage 1 as the starting point for Stage 2 of our optimization framework. However, this framework is flexible, and any other suitable district map that satisfies the legal requirements – such as the grid map released by the IRC (Arizona IRC, 2021) – could also be used as a starting point to this optimization algorithm.

For the district maps in this report, we choose three objectives to optimize: (i) compactness as measured by the sum of district perimeters, (ii) competitiveness as measured by the maximum vote spread, and (iii) competitiveness as measured by the number of competitive districts. Recall that the *vote spread* of a district is the fractional difference between the Democratic and Republican voters in that district. A district is *competitive* if its vote spread is less than 7%.

To optimize for compactness, the local search method (DeFord, Duchin, & Solomon, 2019) is run for 10,000 steps. To optimize for the competitiveness objectives, the same local search method is first run for 10,000 steps to optimize for the corresponding competitive objective. The resulting map may be highly competitive, but may not be compact. To improve the compactness of the map, we optimize the map for compactness while allowing a small decrease in the competitiveness value. The resulting maps are highly competitive and reasonably compact.

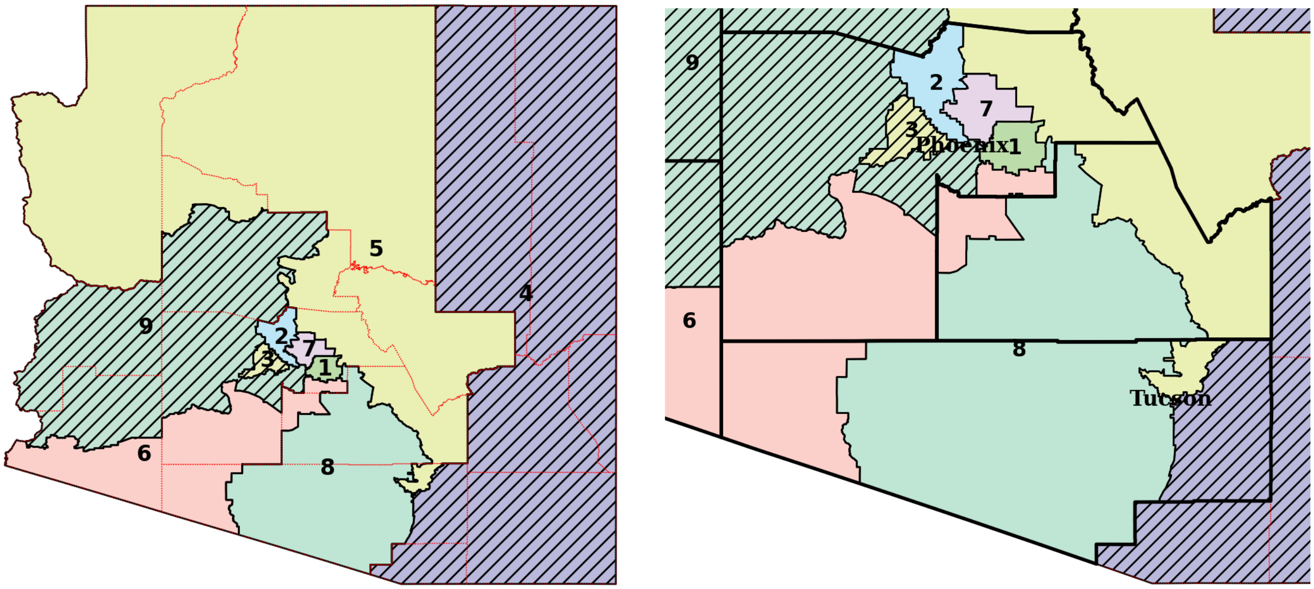
4. District Maps

This section presents a collection of six draft congressional district maps following Arizona's constitutional requirements discussed in Section 2.1 using the optimization method described in Section 4. Each of the six maps labeled A through F emphasizes different scenarios based on the constitutional requirements. Table 1 provides a summary of the district maps. The population deviation tolerance is set to 0.5% for maps A-C and 0.1% for maps D-F. Each map is optimized using a combination of the three redistricting objectives listed in Section 2.2. In particular, maps A, B, D, and E emphasize the two definitions of competitiveness, whereas maps C and F emphasize compactness. All six draft maps perform better than the current congressional district map in terms of Arizona's constitutional requirements, with the exception of population balance. Hence, these maps demonstrate the inherent trade-offs between the constitutional requirements.

Map	**Max. population deviation from ideal (%)**	**No. of divided counties (out of 15)**	**No. of divided census tracts (out of 1,765)**	**No. of majority-minority districts**	**Max. Vote spread (%)**	**No. of competitive* districts**	**No. of safe* Dem. vs. Rep. districts**	**Compactness (km)**
A (Competitive)	0.48%	6	0	3	6.99%	9	0 D, 0 R	6,404
B (Competitive)	0.48%	6	0	4	14.84%	6	0 D, 3 R	4,818
C (Compact)	0.44%	6	0	3	26.29%	3	2 D, 4 R	3,933
D (Competitive)	0.08%	6	0	3	7.46%	5	0 D, 4 R	6,633
E (Competitive)	0.09%	6	0	4	14.12%	8	0 D, 1 R	5,610
F (Compact)	0.09%	6	0	4	25.86%	2	3 D, 4 R	4,575
117th Congressional map**	0.00%	7	151***	3	36.45%	1	4 D, 4 R	6,614

*Table 1. An overview of the district maps. *A district is competitive (vs. safe) if the difference between the Democratic and Republican voters is within (vs. more than) 7% of the total voters in the district. **The 117th Congressional district map (2021-2023) was drawn in 2013. ***Out of 1,526 census tracts based on the 2010 census.*

District map A: Competitive district map with a maximum vote spread of 6.99% and within a 0.5% population deviation



(a) District map A.

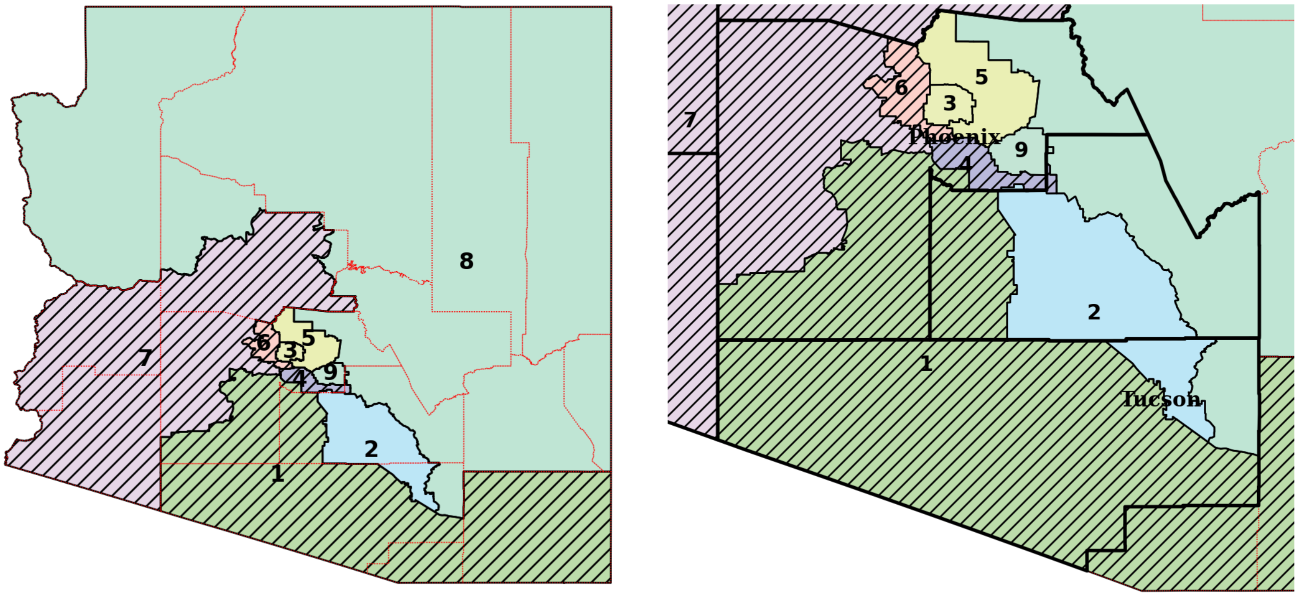
(b) Detail of district map A highlighting Maricopa, Pima and Pinal counties.

Figure 5. A competitive map (A) with a maximum vote spread of 6.99% and within a 0.5% population deviation. The 3 shaded districts are majority-minority districts.

District	Total population			Racial Demographics			Competitiveness			
	Population	Deviation from ideal	Deviation (%)	Hispanic /Latino	NH White	Other Minority races	Vote Spread	Dem. Wins*	Rep. Wins*	Party with a simple majority**
1	790,766	3845	0.48%	25.1%	59.0%	16.0%	5.49%	3	6	R
2	795,317	706	0.09%	29.6%	54.0%	16.3%	6.99%	2	7	R
3	795,798	1187	0.15%	49.2%	36.7%	14.1%	0.85%	6	3	D
4	794,666	55	0.01%	32.7%	45.1%	22.1%	3.36%	7	2	D
5	793,117	1495	0.19%	19.9%	64.5%	15.6%	6.05%	1	8	R
6	795,552	941	0.12%	31.2%	50.9%	17.9%	6.49%	1	8	R
7	793,877	734	0.09%	17.8%	68.4%	13.8%	6.85%	2	7	R
8	794,320	291	0.04%	32.9%	54.8%	12.3%	6.54%	0	9	R
9	798,090	3479	0.44%	37.4%	47.1%	15.5%	5.94%	3	6	R

*Table 2. The demographics for the nine districts in district map A. The vote spreads of the (highly) competitive districts are highlighted in (dark) green, and the NH White populations of the majority-minority districts are highlighted in red. *The number of elections among the 9 elections that each party has a majority in each district. **The party that has a simple majority in an average election.*

District map B: Competitive district map with 6 competitive districts and within a 0.5% population deviation



(a) District map B.

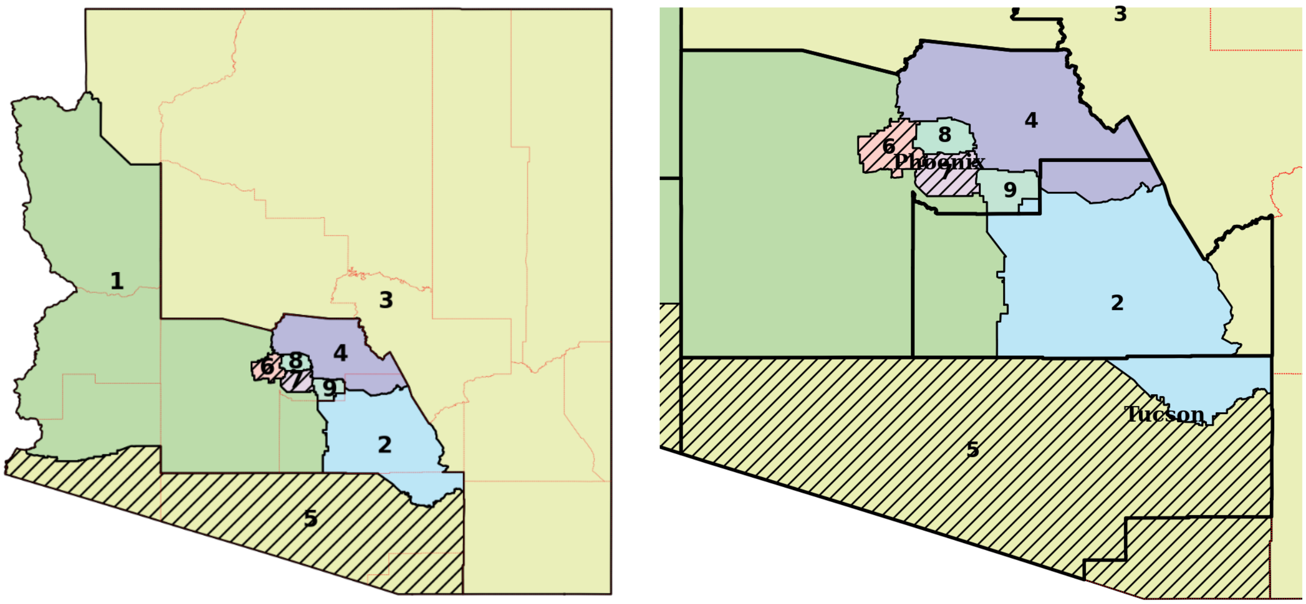
(b) Detail of district map B highlighting Maricopa, Pima and Pinal counties.

Figure 6. A competitive map (B) with 6 competitive districts and within a 0.5% population deviation. The 4 shaded districts are majority-minority districts. The county boundaries are highlighted in red.

District	Total population			Racial Demographics			Competitiveness			
	Population	Deviation from ideal	Deviation (%)	Hispanic /Latino	NH White	Other Minority races	Vote Spread	Dem. Wins	Rep. Wins	Average majority party
1	796,067	1456	0.18%	44.0%	42.6%	13.4%	6.78%	7	2	D
2	798,191	3580	0.45%	28.6%	57.2%	14.2%	4.62%	7	2	D
3	792,984	1627	0.20%	31.8%	51.6%	16.5%	1.95%	4	5	R
4	794,800	189	0.02%	29.7%	48.7%	21.6%	5.67%	6	3	D
5	797,096	2485	0.31%	18.7%	66.0%	15.2%	6.02%	2	7	R
6	790,782	3829	0.48%	41.2%	46.2%	12.6%	6.32%	2	7	R
7	795,569	958	0.12%	41.2%	47.3%	11.5%	14.41%	0	9	R
8	791,076	3535	0.44%	15.8%	59.3%	24.8%	14.84%	0	9	R
9	794,937	326	0.04%	24.8%	61.3%	13.9%	14.16%	0	9	R

*Table 3. The demographics for the nine districts in district map B. The vote spreads of the (highly) competitive districts are highlighted in (dark) green, and the NH White populations of the majority-minority districts are highlighted in red. *The number of elections among the 9 elections that each party has a majority in each district. **The party that has a simple majority in an average election.*

District map C: Compact district map within a 0.5% population deviation



(a) District map C.

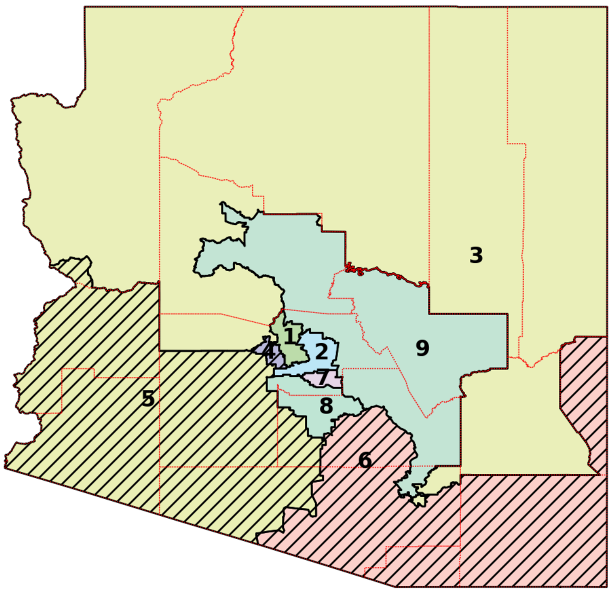
(b) Detail of district map C highlighting Maricopa, Pima and Pinal counties.

Figure 7. A compact district map (C) within a maximum population deviation of 0.5%. The 3 shaded districts are majority-minority districts.

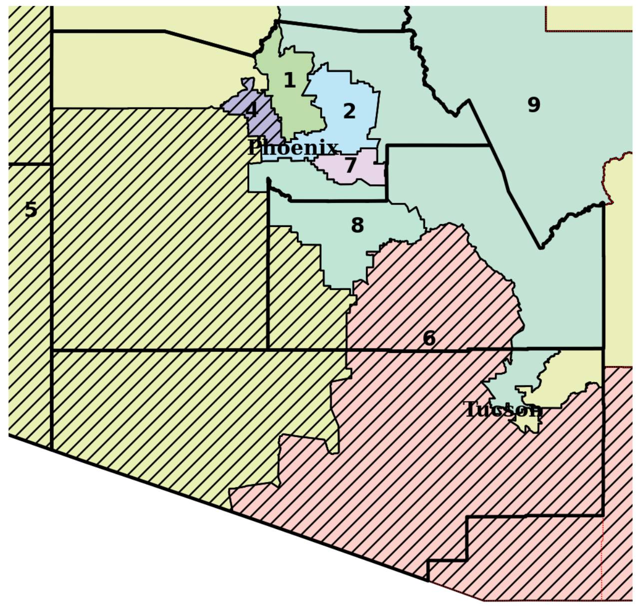
District	Total population			Racial Demographics			Competitiveness			
	Population	Deviation from ideal	Deviation (%)	Hispanic /Latino	NH White	Other Minority races	Vote Spread	Dem. Wins*	Rep. Wins*	Party with a simple majority**
1	794,960	349	0.04%	24.2%	62.1%	13.7%	26.29%	0	9	R
2	795,692	1081	0.14%	25.1%	61.3%	13.6%	1.05%	6	3	D
3	794,664	53	0.01%	17.9%	57.0%	25.1%	11.84%	0	9	R
4	796,743	2132	0.27%	13.5%	74.0%	12.5%	22.06%	0	9	R
5	795,374	763	0.10%	53.5%	36.1%	10.4%	14.15%	9	0	D
6	796,705	2094	0.26%	49.2%	36.2%	14.6%	1.46%	6	3	D
7	794,525	86	0.01%	37.9%	41.4%	20.8%	22.66%	8	1	D
8	791,132	3479	0.44%	30.3%	53.3%	16.5%	1.20%	5	4	D
9	791,707	2904	0.37%	24.5%	58.9%	16.6%	10.59%	0	9	R

*Table 4. The demographics for the nine districts in district map C. The vote spreads of the (highly) competitive districts are highlighted in (dark) green, and the NH White populations of the majority-minority districts are highlighted in red. *The number of elections among the 9 elections that each party has a majority in each district. **The party that has a simple majority in an average election.*

District map D: Competitive district map with a maximum vote spread of 7.46% and within a 0.1% population deviation



(a) District map D.



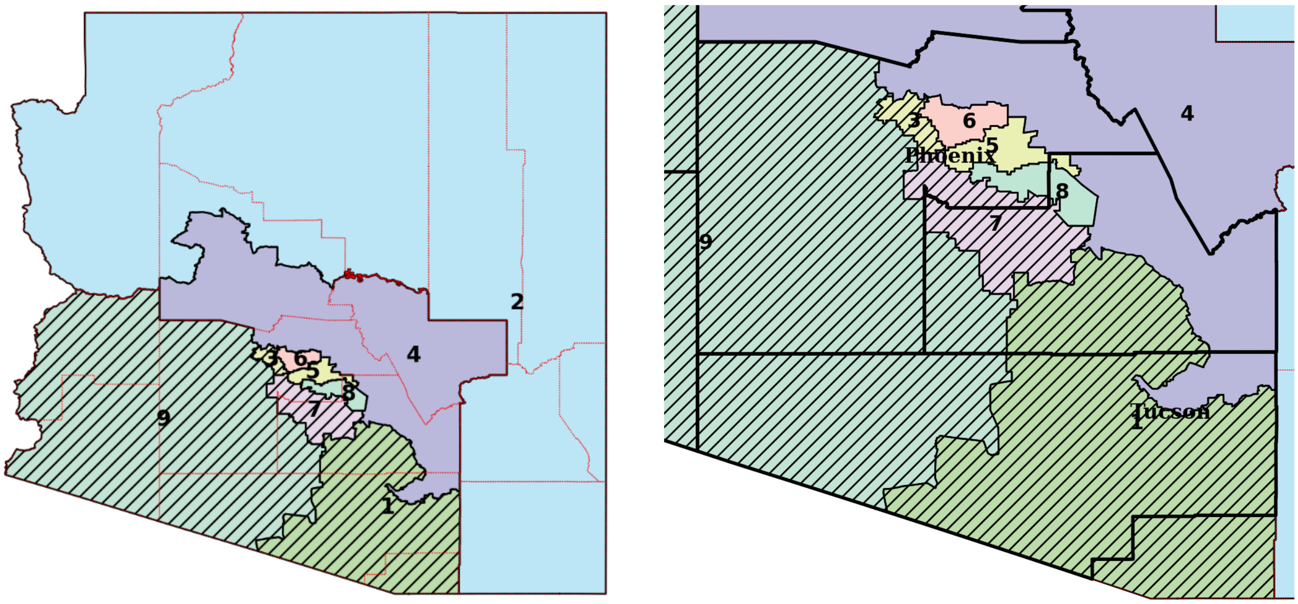
(b) Detail of district map D highlighting Maricopa, Pima and Pinal counties.

Figure 8. A competitive district map (D) with a maximum vote spread of 7.46% and within maximum population deviation of 0.1%. The 3 shaded districts are majority-minority districts.

District	Total population			Racial Demographics			Competitiveness			
	Population	Deviation from ideal	Deviation (%)	Hispanic /Latino	NH White	Other Minority races	Vote Spread	Dem. Wins*	Rep. Wins*	Party with a simple majority**
1	793,941	670	0.08%	23.3%	60.4%	16.3%	5.97%	3	6	R
2	794,042	569	0.07%	29.3%	55.7%	15.0%	0.75%	5	4	R
3	794,714	103	0.01%	16.9%	57.5%	25.6%	7.46%	0	9	R
4	794,358	253	0.03%	39.4%	47.4%	13.2%	7.27%	2	7	R
5	795,131	520	0.07%	48.1%	38.4%	13.5%	7.02%	0	9	R
6	794,918	307	0.04%	43.8%	45.1%	11.0%	2.68%	6	3	D
7	795,216	605	0.08%	25.9%	57.5%	16.6%	5.00%	3	6	R
8	794,277	334	0.04%	27.1%	51.4%	21.4%	3.01%	3	6	R
9	794,905	294	0.04%	21.9%	66.8%	11.2%	7.41%	0	9	R

*Table 5. The demographics for the nine districts in district map D. The vote spreads of the (highly) competitive districts are highlighted in (dark) green, and the NH White populations of the majority-minority districts are highlighted in red. *The number of elections among the 9 elections that each party has a majority in each district. **The party that has a simple majority in an average election.*

District map E: Competitive district map with 8 competitive districts and within a 0.1% population deviation



(a) District map E.

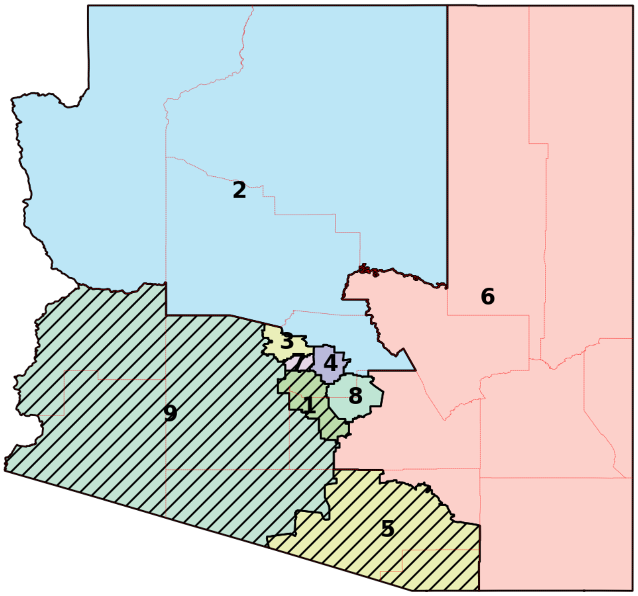
(b) Detail of district map E highlighting Maricopa, Pima and Pinal counties.

Figure 9. A competitive map (E) with 8 competitive districts and within a 0.1% population deviation. The 4 shaded districts are majority-minority districts.

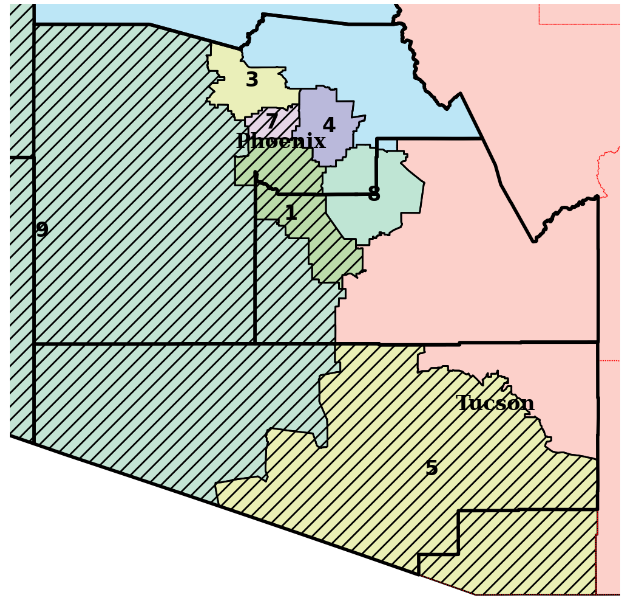
District	**Total population**			**Racial Demographics**			**Competitiveness**			
	Population	**Deviation from ideal**	**Deviation (%)**	**Hispanic /Latino**	**NH White**	**Other Minority races**	**Vote Spread**	**Dem. Wins***	**Rep. Wins***	**Party with a simple majority****
1	795,136	525	0.07%	42.6%	46.1%	11.3%	6.11%	7	2	D
2	794,737	126	0.02%	18.2%	57.3%	24.5%	14.12%	0	9	R
3	794,919	308	0.04%	41.2%	45.4%	13.4%	6.46%	2	7	R
4	793,951	660	0.08%	19.5%	68.0%	12.6%	6.96%	0	9	R
5	794,442	169	0.02%	30.6%	54.8%	14.6%	0.55%	5	4	R
6	795,346	735	0.09%	22.0%	62.0%	16.0%	4.39%	3	6	R
7	794,295	316	0.04%	32.5%	47.1%	20.5%	1.50%	5	4	R
8	794,251	360	0.05%	22.2%	60.5%	17.2%	6.53%	2	7	R
9	794,425	186	0.02%	47.1%	39.2%	13.7%	6.34%	0	9	R

*Table 6. The demographics for the nine districts in district map E. The vote spreads of the (highly) competitive districts are highlighted in (dark) green, and the NH White populations of the majority-minority districts are highlighted in red. *The number of elections among the 9 elections that each party has a majority in each district. ** The party that has a simple majority in an average election.*

District map F: Compact district map within a 0.1% population deviation



(a) District map F.



(b) Detail of district map F highlighting Maricopa, Pima and Pinal counties.

Figure 10. A compact district map (F) within a maximum population deviation of 0.1%. The 4 shaded districts are majority-minority districts.

District	Total population			Racial Demographics			Competitiveness			
	Population	Deviation from ideal	Deviation (%)	Hispanic /Latino	NH White	Other Minority races	Vote Spread	Dem. Wins*	Rep. Wins*	Party with a simple majority**
1	795,351	740	0.09%	39.1%	38.7%	22.2%	14.84%	7	2	D
2	794,822	211	0.03%	13.7%	73.5%	12.8%	25.86%	0	9	R
3	794,253	358	0.05%	19.1%	67.2%	13.7%	15.40%	0	9	R
4	794,451	160	0.02%	22.4%	62.7%	14.9%	6.24%	2	7	R
5	794,762	151	0.02%	44.6%	42.9%	12.5%	23.80%	9	0	D
6	794,667	56	0.01%	22.1%	55.4%	22.5%	9.07%	0	9	R
7	794,088	523	0.07%	48.5%	35.6%	15.9%	15.04%	7	2	D
8	794,683	72	0.01%	19.3%	65.0%	15.6%	19.15%	0	9	R
9	794,425	186	0.02%	47.1%	39.2%	13.7%	6.34%	0	9	R

*Table 7. The demographics for the nine districts in district map F. The vote spreads of the (highly) competitive districts are highlighted in (dark) green, and the NH White populations of the majority-minority districts are highlighted in red. *The number of elections among the 9 elections that each party has a majority in each district. ** The party that has a simple majority in an average election.*

5. Discussion

In this section, we discuss each of the maps presented in Section 4. In Sections 5.1 and 5.2, we focus on how the voters for the two parties are geographically divided among the nine districts with a particular focus on the voters in and around Phoenix and Tucson. In Section 5.3, we derive key insights from adopting Arizona's redistricting requirements in creating these maps.

5.1. Competitive maps (A, B, D and E)

Maps A, B, D and E emphasize competitiveness. Maps A and B have a maximum population deviation of 0.5%, while maps D and E have a maximum population deviation of 0.1%.

Map A is the most competitive, but also the least compact map among the six maps presented in this report. It has all nine districts being competitive with a maximum vote spread of 6.99%. The compactness perimeter is 6,404 km. This high level of competitiveness is achieved by evenly dividing the Democratic voters in Phoenix and Tucson with the Republican voters dispersed in the rest of the state. In particular, Phoenix is divided among six different districts (labeled 1, 2, 3, 6, 7, 9), and Tucson is divided among three remaining districts (labeled 4, 5, 8). Moreover, four of the districts sharing the Phoenix area (labeled 1, 2, 3, 7) are contained entirely within Maricopa County, effectively dividing the Democratic voters in urban Phoenix and the Republican voters in suburban Phoenix. Among the six maps presented in this report, map A is also the least compact with a perimeter of 6,404 km.

Map B is less competitive than map A, but it is more compact. Here, six of the nine districts are competitive with a maximum vote spread of 14.84%. The compactness perimeter is 4,818 km. Hence, maps A and B illustrate a trade-off between competitiveness and compactness. Similar to map A, all the districts share parts of Phoenix and Tucson, although three uncompetitive districts (labeled 7, 8, 9) have a higher fraction of Republicans than Democrats. In particular, six districts (labeled 3, 4, 5, 6, 7, 9) share parts of Phoenix, while three districts (labeled 1, 2, 8) share parts of Tucson. Further, three districts (labeled 3, 5, 6) are contained entirely inside Maricopa County.

Map D has a smaller population deviation than maps A and B, and it is relatively close to being as competitive as map A. Five of the nine districts are competitive with a maximum vote spread of 7.46%. Even though three districts are technically considered uncompetitive (since their vote spread is above 7%), we can see that they are very close to being competitive. This map also has a high compactness perimeter of 6,633 km. Similar to maps A and B, all the districts share parts of Phoenix and Tucson. Six districts (labeled 1, 2, 4, 5, 7, 8) share parts of Phoenix and three districts (labeled 3, 6, 9) share parts of Tucson. Four districts (labeled 1, 2, 4, 7) are contained entirely inside Maricopa County.

Map E is more competitive than map D in terms of the number of competitive districts, but less competitive in terms of the maximum vote spread. Here, eight of the nine districts are competitive with a maximum vote spread of 14.12% (in the one uncompetitive district). This map is also more compact than map D with a perimeter of 5,610 km. While the competitive districts share parts of Phoenix (labeled 3, 5, 6, 7, 8, 9) and Tucson (labeled 1, 4), the uncompetitive district (labeled 2) does not share any part in either city. Instead, it is composed of seven unbroken counties and a portion of Yavapai County, thereby making it a Republican safe district. Three districts (labeled 3, 5, 6) are contained entirely inside Maricopa County.

5.2. Compact maps (C and F)

Maps C and F are compact maps with maximum population deviations of 0.5% and 0.1%, respectively. These district maps are created without using any voting data.

Map C is the most compact map found by the algorithm with a perimeter of 3,933 km. It has three competitive districts and a maximum vote spread of 26.29%. Among the six uncompetitive districts, two districts have Democratic majorities and four have Republican majorities. Six districts (labeled 1, 4, 6, 7, 8, 9) share parts of Phoenix. Of these, district 7 (with vote spread 22.66%) packs the Democratic voters in the city of Phoenix, while districts 4 and 9 (with respective vote spreads 22.06% and 10.59%) pack the Republican voters in the region surrounding Phoenix. Districts 6 and 8 (with respective vote spreads 1.46% and 1.20%) are highly competitive with a roughly equal share of both party voters. Districts 2 and 5 share parts of Tucson, where district 2 (with a vote spread of 1.05%) is a competitive district, while district 5 (with a vote spread of 14.15%) packs the Democratic voters. District 3 (with vote spread 11.84%) shares parts of neither city and has a Republican majority.

Map F is the most compact map found by the algorithm for a smaller population deviation tolerance of 0.1% It has a perimeter of 4,575 km. It has two competitive districts and a maximum vote spread of 25.86%. Among the other seven districts, three districts have Democratic majorities and four have Republican majorities. Six districts (labeled 1, 3, 4, 7, 8, 9) share parts of Phoenix. Of these, districts 1 and 7 (with respective vote spreads 14.84% and 15.07%) pack the Democratic voters in the city of Phoenix, while districts 3 and 8 (with respective vote spreads 15.40% and 19.15%) pack the Republican voters in the region surrounding Phoenix. Districts 4 and 9 (with respective vote spreads 6.24% and 6.34%) are competitive. Districts 5 and 6 share parts of Tucson, where district 5 (with a vote spread of 23.80%) packs the Democratic voters in Tucson, whereas district 6 (with a vote spread of 9.07%) has a Republican majority. District 2 (with vote spread 25.86%) overlaps neither Phoenix nor Tucson and has a Republican majority.

5.3. Key insights

The six draft congressional district maps presented in this report emphasize different aspects of Arizona's constitutional requirements such as population deviation, competitiveness, and compactness. These maps illustrate the trade-offs between these requirements. In this section, we discuss some key insights that can be drawn from these maps.

Compactness vs Competitiveness: The most compact maps generated by the algorithm (i.e., maps C and F) pack the Democratic voters in the urban centers, resulting in highly uncompetitive (or safe) districts. Due to this, both the maps give Republicans a partisan advantage. On the other hand, the highly competitive district maps (i.e., maps A, B, D and F) join Democratic voters in the urban centers with Republican voters outside of them to evenly divide the voters. This tends to produce noncompact districts.

The effect of population balance threshold: Comparing the maps A-C and maps D-F, it is evident that setting a smaller threshold for population deviation restricts the ability of the algorithm to create districts that are both competitive and compact. This highlights the trade-off between extremely tight population balance and other Arizona redistricting requirements.

Partisan fairness: Even though the Arizona Constitution does not emphasize partisan fairness as a redistricting criteria, we now discuss how the maps generated by the algorithm perform with respect to the partisan fairness measures discussed in Section 2.2. Recall that *efficiency gap* (EG) measures the difference in wasted votes between the two parties, and *partisan asymmetry* (PA) measures the difference in the rate at which the two parties gain or lose seats when there are fluctuations in their share of voters. Table 8 presents the partisan fairness values for our presented district maps and for the 117th Congressional district map.

Plan	Efficiency Gap	Partisan Asymmetry
A	24.1%	1.82%
B	12.3%	0.45%
C	6.4%	1.67%
D	34.4%	1.19%
E	33.8%	0.80%
F	16.1%	2.71%
117th Congressional map*	2.5%	2.23%

*Table 8. Partisan fairness metrics for the presented maps. *The 117th Congressional district map (2021-2023) was drawn in 2013. The cells highlighted green indicate relatively low scores for either of the metrics.*

The competitive maps (A, B, D and E) have high values of EG. The computation of EG solely relies on which party has a simple majority in each district. However, the creation of competitive districts ignores which party has a simple majority in each district. In a competitive district map, when Republicans have a simple majorities in a disproportionate number of districts, the Democrats waste disproportionately more voters than Republicans, thereby producing a high value of EG.

The compact map F also has a high value of EG. This is attributed to the packing of urban Democratic voters in compact districts, which leads Democrats to waste significantly more voters than Republicans, giving rise to a high value of EG. Despite this effect, map C is within the 8% threshold set by the creators of EG below which a map may be considered “fair” (Stephanopoulos

& McGhee, 2015). The EG value for the 117th Congressional district map is relatively low since it has proportional simple majorities for the two parties (i.e., five Republican and four Democratic districts), even though the districts are highly uncompetitive with a maximum vote spread of 36.45%.

On the other hand, competitive maps A, B, D and E have relatively small values of PA. This indicates that competitiveness correlates with partisan symmetry, which is a positive indicator that the Arizona Constitution's emphasis on competitiveness aligns with the symmetry standard. The compact maps C and F have mixed effect on PA, and the relationship between compactness and PA is unclear. The 117th Congressional district map has a relatively high value of PA when compared to competitive maps A, B, D, and E, which indicates that with fluctuations in voters, one party could receive disproportionately more advantage than the other.

In conclusion, the maps and the analysis in this report demonstrate the potential of computational redistricting in Arizona. The trade-off between the criteria must be carefully considered in designing an algorithmic approach that seeks to optimize for certain criteria. The draft congressional district maps in this report illustrate the capabilities of our optimization algorithm. When the IRC finishes collecting communities of interest, our algorithm can preserve those communities while balancing trade-offs between the other redistricting requirements to produce transparent district maps that best serve Arizonans.

References

Altman, M., & McDonald, M. (2018). Redistricting by Formula: An Ohio Reform Experiment. *American Politics Research, 46*(1), 103-131.

Arizona. (2021). *Arizona State Constitution, Article 4, Part 2, Section 1*. Retrieved October 1, 2021, from https://www.azleg.gov/const/4/1.p2.htm

Arizona IRC. (2021). *Grid Map*. Retrieved October 1, 2021, from https://redistricting-irc-az.hub.arcgis.com/pages/grid-map

Ballotpedia. (2021). *Majority-minority districts*. Retrieved from Ballotpedia: https://ballotpedia.org/Majority-minority_districts

Bernstein, M., & Duchin, M. (2017). A Formula Goes to Court: Partisan Gerrymandering and the Efficiency Gap. *Notices of the American Mathematical Society, 64*(09), 1020-1024.

Bozkaya, B., Erkut, E., & Laporte, G. (2003). A tabu search heuristic and adaptive memory procedure for political districting. *European journal of operational research, 144*(1), 12-26.

Chambers, C. P., Miller, A. D., & Sobel, J. (2017). Flaws in the Efficiency Gap. *Journal of Law & Politics, 33*.

Cho, W. K. (2017). Measuring Partisan Fairness: How Well Does the Efficiency Gap Guard against Sophisticated as Well as Simple-Minded Modes of Partisan Discrimination. *University of Pennsylvania Law Review Online, 166*.

DeFord, D., Dhamankar, N., Duchin, M., Gupta, V., McPike, M., Schoenbach, G., & Sim, K. W. (2021). Implementing partisan symmetry: Problems and paradoxes. *arXiv pre-print*.

DeFord, D., Duchin, M., & Solomon, J. (2019). Recombination: A family of Markov chains for redistricting. *arXiv preprint arXiv:1911.05725*.

DeFord, D., Duchin, M., & Solomon, J. (2020). A Computational Approach to Measuring Vote Elasticity and Competitiveness. *Statistics and Public Policy, 7*(1), 69-86. doi:10.1080/2330443X.2020.1777915

Duchin, M. (2018). *Outlier Analysis for Pennsylvania Congressional Redistricting.* Technical Report.

Grofman, B. (1983). Measures of Bias and Proportionality in Seats-Votes Relationships. *Political Methodology, 9*(3), 295-327.

Grofman, B., & Cervas, J. (2021). Recent Approaches to the Definition and Measurement of Compactness. *SSRN*. Retrieved from https://ssrn.com/abstract=3919249

Grofman, B., & King, G. (2007). The Future of Partisan Symmetry as a Judicial Test for Partisan Gerrymandering after LULAC v. Perry. *Election Law Journal: Rules, Politics, and Policy, 6*(1), 2-35.

Hirano, S., & Snyder Jr., J. M. (2012). What Happens to Incumbents in Scandals? *Quarterly Journal of Political Science, 7*(4), 447-456. doi:10.1561/100.00012039

Katz, J. N., King, G., & Rosenblatt, E. (2020). Theoretical Foundations and Empirical Evaluations of Partisan Fairness in District-Based Democracies. *American Political Science Review, 114*(1), 164-178.

Kaufman, A. R., King, G., & Komisarchik, M. (2021). How to Measure Legislative District Compactness If You Only Know It When You See It. *American Journal of Political Science, 65*(3), 533-550. doi:10.1111/ajps.12603

McCarty, N., Poole, K. T., & Rosenthal, H. (2009). Does Gerrymandering Cause Polarization? *American Journal of Political Science, 53*(3), 666-680. doi:10.1111/j.1540-5907.2009.00393.x

McGhee, E. (2020). Partisan Gerrymandering and Political Science. *Annual Review of Political Science, 23*, 171-185. doi:10.1146/annurev-polisci-060118-045351

NCSL. (2020, January 15). *2010 Redistricting Deviation Table*. Retrieved October 1, 2021, from https://www.ncsl.org/research/redistricting/2010-ncsl-redistricting-deviation-table.aspx

Niemi, R. G., Grofman, B., Carlucci, C., & Hofeller, T. (1990). Measuring Compactness and the Role of a Compactness Standard in a Test for Partisan and Racial Gerrymandering. *Journal of Politics, 52*(4), 1155-1181.

OpenPreincts. (2020). *OpenPreincts*. Retrieved from https://openprecincts.org/az

Ricca, F., Scozzari, A., & Simeone, B. (2013). Political Districting: from classical models to recent approaches. *Ann Oper Res, 204*, 271-299. doi:10.1007/s10479-012-1267-2

Stephanopoulos, N. O., & McGhee, E. M. (2015). Partisan Gerrymandering and the Efficiency Gap. *University of Chicago Law Review, 82*(2), 831-900.

Stephanopoulos, N. O., & McGhee, E. M. (2018). The Measure of a Metric: The Debate over Quantifying Partisan Gerrymandering. *Stanford Law Review, 70*.

Swamy, R., King, D. M., & Jacobson, S. H. (2019). *Multi-Objective Optimization for Political Districting: A Scalable Multilevel Approach.* Optimization Online.

Tapp, K. (2019). Measuring Political Gerrymandering. *The American Mathematical Monthly, 126*(7), 593-609. doi:10.1080/00029890.2019.1609324

U.S. Census Bureau. (2020). Retrieved from https://data.census.gov/cedsci/

Validi, H., Buchanan, A., & Lykhovyd, E. (2021). Imposing contiguity constraints in political districting models. *Operations Research (to appear)*.