This research analysis was conducted as part of the Institute for Computational Redistricting (ICOR) at the University of Illinois at Urbana-Champaign. This activity was conducted in a non-partisan manner, with any political descriptors used reflecting the results of the quantitative analysis, not the opinions of the researchers nor ICOR.
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Executive Summary

The Institute for Computational Redistricting (http://redistricting.cs.illinois.edu) is a research group at the University of Illinois at Urbana-Champaign. Under the direction of Dr. Sheldon H. Jacobson (http://shj.cs.illinois.edu) and Dr. Douglas M. King, the group focuses on computational methods for redistricting to provide transparency within the redistricting process. In January 2021, the Missouri League of Women Voters (MO-LWV) contacted the group to prepare U.S. congressional district plans for Missouri, and state legislative district plans (state senate/house) that satisfy the new redistricting criteria amended to the Missouri Constitution in 2020. The new criteria for state legislative plans introduces a maximum allowed percent deviation for district populations, and prioritizes compactness and the preservation of political subdivisions over partisan fairness requirements. For congressional plans, Missouri only requires contiguous and compact districts, in addition to the requirements from the U.S. Constitution. The goals of this analysis are to examine the physical and partisan characteristics of plans that satisfy constitutional requirements, and to examine the interplay between legal requirements and partisan fairness.

For state legislative district plans, we explore questions such as how many counties (i.e., a type of political subdivision) can be kept intact under the new population requirement, how many seats can each party expect to win, and how much can partisan fairness be improved (beyond any constitutionally required thresholds). For example, the new population requirement affects how many state legislative districts can fit inside highly populated counties. Also, prioritizing compactness and preserving political subdivisions over partisan fairness allows Democrats to be packed into urban districts.

The Missouri Constitution has fewer requirements for congressional district plans than for state legislative district plans. Therefore, in contrast to the state legislative plans, there are many ways to draw the congressional district boundaries that satisfy constitutional requirements; some ways exhibit good partisan fairness, and others exhibit poor partisan fairness. It is also important to create these district plans by transparent means; with no insight into how a plan was constructed, it is not clear whether undesirable qualities (e.g., packing and cracking, non-competitive districts) are due to gerrymandering or are a natural result of constitutional requirements and political geography.
We use an optimization algorithm to create a collection of district plans (two state senate, two state house, and eight congressional) that satisfy legal requirements and prioritize different fairness objectives (e.g., compactness, the Efficiency Gap). Although compactness is prioritized in the Missouri Constitution (for both state legislative and congressional plans), focusing on additional aspects of fairness can illustrate the range of partisan outcomes that is possible for Missouri’s legal requirements and political geography. Optimization algorithms promote transparency in each step of the districting process, since they have clearly defined objectives, constraints, and parameters. A discussion accompanies the collection of district plans, including the advantages and disadvantages of each plan, and how legal requirements and political geography impact the redistricting process.

Ultimately, the results show that Missouri’s unique political geography, combined with the constitutional requirements, gives Republicans an inherent advantage. For example, district plans created to solely prioritize compactness tend to pack/crack Democratic votes, since Democrats are concentrated in a few urban centers with fewer voters throughout the rest of the state. Hence, many Democratic votes are considered wasted, since packed districts have many more votes than are needed to win their elections and cracked districts are lost by small margins. In contrast, Republicans are more evenly distributed throughout the state, and hence tend to waste fewer votes than Democrats do and are better able to translate their votes into seats. Preserving political subdivisions (e.g., county lines), as required by the Missouri Constitution for state legislative district plans, exacerbates this problem. The state legislative plans must satisfy a constitutionally required threshold for partisan fairness (related to the Efficiency Gap), but plans that solely prioritize compactness can satisfy this threshold while still wasting many votes.

We ran experiments to improve partisan fairness in state legislative plans (as measured by various voting-based metrics), but were unable to make improvement while maintaining compact districts and preserving county lines. Hence, Missouri’s constitutional requirements severely limit the options available for legally acceptable state senate/house district plans.

In contrast, partisan fairness can be substantially improved in congressional plans while maintaining compactness and a majority-minority district. We
present compact congressional plans that also have good Efficiency Gap values, good Partisan Asymmetry values, or four competitive districts. In particular, the congressional plans that prioritize the Efficiency Gap and compactness have compactness scores comparable to plans that solely prioritize compactness, but their Efficiency Gap values have improved (i.e., decreased) by more than 15%.
1. Introduction

Missouri has changed its state legislative redistricting requirements twice since the last redistricting cycle in 2011 (Mo. Const. art. III, §3). While the 2018 redistricting amendment prioritized partisan fairness (i.e., the Efficiency Gap), the 2020 redistricting amendment prioritizes compactness and the preservation of political subdivisions (e.g., counties). The 2020 amendment also defines a maximum allowed percent deviation for district populations. For congressional redistricting, Missouri requires contiguous and compact districts, in addition to the requirements from the U.S. Constitution (Mo. Const. art. III, §45). The Missouri League of Women Voters (MO-LWV) contacted the Institute for Computational Redistricting (ICOR) to construct state legislative and congressional district plans that satisfy constitutional requirements.

This report provides the MO-LWV with a collection of two state senate, two state house, and eight congressional district plans. The plans are constructed with an optimization algorithm to promote transparency in the districting process. To assess the preservation of political subdivisions for the state legislative plans, we examine how many districts can fit inside highly populated counties under the new population requirement, and how many counties must be split. For the congressional plans, we maintain a majority-minority district. To examine the interplay between redistricting requirements and partisan fairness, some plans presented in this report prioritize compactness, while others prioritize partisan aspects of fairness (e.g., the Efficiency Gap). Although the Missouri Constitution prioritizes compactness, focusing on multiple aspects of fairness allows one to examine how constitutional requirements and political geography affect the level of political fairness achievable for Missouri. Each plan is scored with various fairness metrics, and the advantages and disadvantages between plans are discussed. The results show that state legislative district plans that satisfy constitutional requirements have an inherent Republican advantage, likely attributable to the political geography of the state. For congressional district plans, the results show that compactness and good political fairness are achievable for Missouri.

This report is organized as follows. Section 2 describes the fairness metrics used to evaluate district plans. Section 3 gives an overview of the optimization method used to construct the plans that optimize these fairness metrics. Section 4 outlines Missouri’s redistricting requirements, data, and geography. Lastly, Section 5 presents the district plans and Section 6 discusses and compares their partisan characteristics.
2. Fairness Metrics

In the context of redistricting, “fairness” can be interpreted in a number of ways. Sometimes fairness focuses on political parties; for example, a district plan could be considered fair if neither party is packed and cracked (i.e., concentrated in a few districts where it wins by overwhelming margins, then diluted among the remaining districts). Fairness can also mean reasonably shaped districts, or competitive districts. Throughout this report, we refer to metrics that use voting data as partisan fairness metrics; compactness is the only fairness metric considered in this report that is not a partisan fairness metric. Each district plan presented in this report is evaluated with various metrics that quantify these common aspects of fairness. This section gives an overview of each metric.

- **Compactness:** A district is compact if it has a simple shape (such as a circle or square, as in Figure 1b), as opposed to a convoluted shape (as in Figure 1a). Simple shapes are preferred, since convoluted district shapes can be a result of intentional boundary manipulation for political gain. For example, the salamander-shaped Massachusetts district that inspired the term gerrymander was constructed to pack Federalist voters (Massachusetts Historical Society). There are several ways one could quantify district compactness (Young 1988); we choose to measure compactness as the sum of all district perimeters (reported in miles). We exclude perimeter segments that coincide with Missouri’s state boundary, since the state boundary will always overlap with district perimeter segments for any district plan. A district with an irregular shape (as in Figure 1a) has a larger perimeter than a district with a simpler shape (as in Figure 1b), which means that smaller values of this metric indicate more compact districts.

(a) A non-compact district with a perimeter of 197 miles
(b) A compact district with a perimeter of 94 miles

*Figure 1* Two examples of district shapes (for roughly the same location and district population). Figure 1a shows a non-compact district and Figure 1b shows a compact district.
- **Efficiency Gap**: The Efficiency Gap aims to quantify packing and cracking. A packed party wastes votes because it wins districts by overwhelming margins; a cracked party wastes votes because it narrowly loses many districts. Hence, “wasted votes” are votes cast for a district’s losing party, or votes cast for a district’s winning party in excess of the 50% needed to win the election. The Efficiency Gap measures the difference in wasted votes between both parties and reports this difference as a percentage of the total votes cast for these two parties (Stephanopoulos and McGhee 2015). Smaller values of this metric indicate that both parties waste a similar number of votes, which means both parties are packed and cracked to a similar degree. For example, the vote-share scenario in Figure 2a has a large Efficiency Gap because Republicans are cracked, and therefore waste many votes; the vote-share scenarios in Figures 2b, 2c, and 2d have small Efficiency Gap values, because both parties waste votes equally.

- **Shifted Efficiency Gap**: The Shifted Efficiency Gap (referred to as “Competitiveness” in the Missouri Constitution (Mo. Const. art. III, §3)) examines how the Efficiency Gap changes with small shifts in voter preferences. The Efficiency Gap relies on voting data from previous elections, and voting behavior might change in future elections. Consider a scenario with multiple very competitive districts (as in Figure 2c); Democrats might narrowly win all the districts in one election, then narrowly lose them all in the next election. Democrats waste many more votes for the latter outcome than the former, even though the change in voter preference is small. To capture the robustness of a district plan, the Shifted Efficiency Gap calculates the maximum Efficiency Gap value for scenarios in which the statewide vote-share shifts 1-5% in favor of either party (uniformly among all districts). Smaller values for this metric indicate that both parties waste a similar number of votes, even with small shifts in voter preference. For example, the vote-share scenario in Figure 2b has a better Shifted Efficiency Gap than Figure 2c because the seat outcome does not dramatically change with small vote-share shifts. For a more technical analysis of the Shifted Efficiency Gap and its potential flaws, see DeFord et al. (2020).

- **Partisan Asymmetry**: Partisan Asymmetry observes to what extent both parties receive different seat outcomes for the same vote-share scenarios (Grofman and King 2007). Similar to the Shifted Efficiency Gap, Partisan Asymmetry compares the rate at which both parties win/lose seats as the statewide vote-share shifts uniformly among
(a) Democrats win five seats and Republicans win one seat. Five seats are competitive. The Efficiency Gap is bad because Republicans are cracked. The Shifted Efficiency Gap is bad because small changes in vote-share could dramatically alter the seat outcome. Partisan Asymmetry is good because the district vote-shares are mostly symmetrically distributed.

(b) Democrats and Republicans each win three seats. No seats are competitive. The Efficiency Gap is good because both parties waste votes equally. The Shifted Efficiency Gap is good because increases/decreases in vote-share for either party do not dramatically alter the seat outcome. Partisan Asymmetry is good because the district vote-shares are symmetrically distributed.

(c) Democrats and Republicans each win three seats. All seats are competitive. The Efficiency Gap is good because both parties waste votes equally. The Shifted Efficiency Gap is bad because small changes in vote-share could dramatically alter the seat outcome. Partisan Asymmetry is good because the district vote-shares are symmetrically distributed.

(d) Democrats and Republicans each win three seats. Two seats are competitive. The Efficiency Gap is good because both parties waste votes equally. The Shifted Efficiency Gap is bad because small changes in vote-share could either change the seat outcome or indicate packing/cracking. Partisan Asymmetry is bad because the district vote-shares are asymmetrically distributed.

Figure 2  Example vote-share scenarios for six districts. Although the overall vote-share is split 50-50 (assuming equal turnout in all districts), there are a number of different ways the voters can be grouped into districts. Democratic fractions are shown on the bottom in blue and Republican fractions are shown on the top in red.

all districts. As the vote-share for one party gradually increases to 100% or decreases to 0%, we can observe how many seats that party would hypothetically win for each of those vote-share scenarios. For example, Figure 3 shows the number of seats each party would hypothetically win for vote-shares from 0-100% in two example district plans (these plots are called vote-seat curves). The more space exists between the two
curves, the more asymmetry is present in the district plan. For the plan in Figure 3a, if Democrats and Republicans were to both win 50% of the votes, they would win 40% and 60% of the seats, respectively. However, for the plan in Figure 3b, Democrats and Republicans each win 50% of the seats for 50% of the votes. In general, there is a large amount of space between the two curves in Figure 3a, while the curves in Figure 3b are nearly identical. Therefore, the plan in Figure 3b is more symmetric than the plan in Figure 3a. The Partisan Asymmetry metric value is calculated as the area between both parties’ vote-seat curves (Grofman 1983). This metric typically takes on values between 0.00 and 0.11. The minimum value of zero occurs when the vote-seat curves are exactly the same; however, sometimes factors such as political geography or number of districts can prevent a state from achieving a value of zero. Similarly, the largest value achievable varies slightly from state to state. As a rough guideline, smaller values up to 0.01 indicate very symmetric plans, while larger values such as 0.08-0.11 indicate very asymmetric plans.

As additional examples, the vote-share scenarios in Figures 2b and 2c have small Partisan Asymmetry values because the vote-shares are symmetrically distributed (i.e., both parties are spread evenly across the districts). The vote-share scenario in Figure 2d has a large Partisan Asymmetry value because the vote-shares are asymmetrically distributed.

![Figure 3](image.png)  
Figure 3: Vote-seat curves for Democrats and Republicans. The curves in Figure 3a are for an asymmetric district plan and the curves in Figure 3b are for a symmetric district plan.

- **Competitiveness:** Maintaining competitive districts can encourage voter turnout, reduce district packing, and discourage candidate complacency (Hirano and Snyder
To assess the competitiveness of a district plan, we display the Democrat/Republican vote-share in each district and report the number of districts within a 10% margin of victory. For example, the vote-shares scenarios in Figures 2a and 2c have many competitive districts, while the scenario in Figure 2b has no competitive districts.

3. Optimization Method

This section describes how the optimization algorithm constructs district plans. In general, the algorithm aims to find a district plan with the best fairness metric value, within the constraints of legal requirements. The algorithm is called a local search method, meaning it starts with a given district plan (e.g., the plan currently in place, or another proposed plan) and improves it by making a sequence of small changes to district boundaries. While a single small change might not drastically transform the plan, performing thousands of them can lead to a significant improvement as the algorithm continues to run. The basic steps of the algorithm, based on a method from DeFord et al. (2019), are listed below.

1. Choose an initial district plan, a fairness metric to improve (e.g., compactness, Efficiency Gap), and constraints to enforce (e.g., roughly equal district populations).
2. Randomly choose two neighboring districts.
3. Erase the boundary between these two districts and randomly draw a new boundary that maintains contiguity. Note that this action only affects the two chosen districts.
4. Check whether this new boundary satisfies the constraints chosen in Step 1. If it does not, return to Step 3. If it does, continue to Step 5.
5. Check whether this new boundary improves the chosen fairness metric. If it does not, return to Step 3. If it does, continue to Step 6.
6. Record this boundary. Repeat Steps 3-5 to create a collection of viable new boundaries.
7. From the collection of viable boundaries, select the boundary that yields the greatest improvement in the fairness metric. Update the district plan accordingly. Repeat Steps 2-6 as needed.

Figure 4 shows an example sequence of changes to improve compactness in a four-district plan, using counties as district building blocks. The initial plan (Figure 4a) has convoluted
districts with long tendrils. Each algorithm step makes the boundary between two districts less convoluted. After four steps, there are no tendrils and the districts all have simple shapes (Figure 4e).

Section 4 outlines the redistricting constraints used in this algorithm from the Missouri Constitution. Section 5 briefly discusses how the algorithm steps are applied to construct the district plans, and Appendix A provides additional details for algorithm application (including parameter choices and number of iterations).

![Image showing examples of district changes](image)

Figure 4 Example steps of the optimization algorithm using a four-district plan at the county level. Each step shows which two districts are changed to improve compactness.

4. Redistricting in Missouri
This section describes how state legislative and congressional redistricting is conducted in Missouri. We discuss Missouri’s geography, the data sources used for our experiments, and the redistricting requirements in the Missouri Constitution.

4.1. Missouri Geography
The discussion of district plans in Section 5 includes references to certain Missouri counties and cities by name, so we provide a brief overview here. Missouri has 114 counties and one
independent city (St. Louis City, which we treat as a county in this report); we mainly discuss the most populated counties, since these counties are the most likely to be split between multiple districts. Figure 5 shows where these counties are located within the state. Jackson County and Clay County are in the west; these counties contain most of Kansas City. Greene County is in the south; this county contains the city of Springfield. Boone County is in the center; this county contains the city of Columbia. Franklin County, St. Charles County, Jefferson County, St. Louis County, and St. Louis City are in the east.

Figure 5 Missouri counties. Highly populated counties are highlighted using insets. In the west, there is Jackson County and Clay County (the Kansas City area); in the south, there is Greene County (containing the city of Springfield); in the center, there is Boone County (containing the city of Columbia); in the east, there is Franklin County, St. Charles County, Jefferson County, St. Louis County, and St. Louis City (the St. Louis area).

4.2. Data Sources

Redistricting requires data describing state geography, population, and election results. The data sources used to create district plans for this report are listed below.
• **State geography:** Districts are constructed using geographic units, such as census blocks, census block groups, census tracts, and counties. The U.S. Census Bureau provides spatial data for these units (U.S. Census Bureau 2020b). With spatial data, we can determine which units are neighbors (to enforce district contiguity) and the length of shared borders between neighboring units (to calculate district perimeters for compactness).

• **Population:** In addition to spatial data, the U.S. Census Bureau also provides population counts for geographic units from the 2020 decennial census (U.S. Census Bureau 2020a). Population data are needed to ensure that all districts have roughly equal populations.

• **Demographics:** To comply with the Voting Rights Act of 1965, states typically construct majority-minority districts for congressional district plans (Ballotpedia 2015). To maintain such districts, we use demographic data from the 2020 decennial census (U.S. Census Bureau 2020a).

• **Election results:** While compactness can be calculated with spatial data (i.e., state geography data), the other fairness metrics in Section 2 rely on voting data from past elections. The Missouri Constitution requires the use of voting data averaged from governor, United States Senate, and Presidential races for the past three general elections (2016, 2018, and 2020) (Mo. Const. art. III, §3). These data sets are available at the precinct level on the Harvard Dataverse (Voting and Election Science Team 2021). To construct districts using finer census units (e.g., census block groups), voting data must be disaggregated from the precinct level to the finer level. For this report, we distribute precinct-level voting data proportionally to the block level; from the block level, the voting data are aggregated to block groups or tracts.

With voting data, we can also examine Missouri’s political geography. According to this set of data, Missouri voters are roughly 46.5% Democrat and 53.5% Republican. Figure 6 shows which census blocks lean Democrat and which lean Republican. Democratic voters are concentrated in urban centers (such as Kansas City and St. Louis), while Republicans are spread throughout the state. As we discuss in Section 6, this distribution of voters prevents significant improvement to partisan fairness in state legislative plans.
4.3. Constitutional Requirements

This section excerpts relevant text from the Missouri Constitution describing the requirements for state legislative and congressional redistricting, and discusses how these requirements are reflected and implemented within our algorithm. The plans in this report satisfy these requirements to the extent possible.

4.3.1. State Legislative Redistricting Requirements Below we discuss the new state legislative redistricting requirements amended to the Missouri Constitution in 2020 (Mo. Const. art. III, §3).

- Population balance:

Relevant text from the Missouri Constitution: “Districts shall be as nearly equal as practicable in population, and shall be drawn on the basis of one person, one vote. Districts are as nearly equal as practicable in population if no district deviates by more than one percent from the ideal population of the districts, as measured by dividing the number of districts into the statewide population data being used, except that a district may deviate by up to three percent if necessary to follow political subdivision lines [...]”

While the Missouri Constitution did require nearly equal district populations in 2010, it did not specify a maximum allowed percent deviation. The current state legislative district plans do not satisfy this new population requirement, since some districts have a deviation of 3.5-4%. For example, if the current senate plan were redrawn to satisfy this requirement (using the 2010 population data) Jackson County
would no longer be able to contain four whole senate districts. All state legislative plans in this report have a maximum 3% deviation from the ideal population. The most notable difference in population distribution between 2010 and 2020 is that Boone County (containing the city of Columbia) now has sufficient population for exactly one senate district.

- **No racial discrimination:**

  Relevant text from the Missouri Constitution: “Districts shall be established in a manner so as to comply with all requirements of the United States Constitution and applicable federal laws, including, but not limited to, the Voting Rights Act of 1965 (as amended). The following principles shall take precedence over any other part of this constitution: no district shall be drawn in a manner which results in a denial or abridgement of the right of any citizen of the United States to vote on account of race or color; and no district shall be drawn such that members of any community of citizens protected by the preceding clause have less opportunity than other members of the electorate to participate in the political process and to elect representatives of their choice.”

  The MO-LWV recommended creating as many competitive districts as possible to avoid racial discrimination, so that minority voters are not packed into safe Democratic districts. As noted in Section 5, we were unable to increase the number of competitive seats while maintaining compactness, preserving the same number of whole counties, and satisfying the partisan fairness threshold (explained later in this section). Although we do not incorporate racial data into the algorithm for state legislative plans, Section 6 lists the number of majority-minority districts for each state senate/house plan (defined as a district in which the non-Hispanic white population accounts for less than 50% of the total district population).

- **Contiguity and compactness:**

  Relevant text from the Missouri Constitution: “Subject to the requirements of [population balance and no racial discrimination], districts shall be composed of contiguous territory as compact as may be. Areas which meet only at the points of adjoining corners are not contiguous. In general, compact districts are those which are square, rectangular, or hexagonal in shape to the extent permitted by natural or political boundaries.”
All state legislative plans in this report are contiguous and optimized for compactness.

- Preservation of political subdivisions:

  Relevant text from the Missouri Constitution: “To the extent consistent with [population balance, no racial discrimination, contiguity, and compactness], communities shall be preserved. Districts shall satisfy this requirement if district lines follow political subdivision lines to the extent possible, using the following criteria, in order of priority. First, each county shall wholly contain as many districts as its population allows. Second, if a county wholly contains one or more districts, the remaining population shall be wholly joined in a single district made up of population from outside the county. If a county does not wholly contain a district, then no more than two segments of a county shall be combined with an adjoining county. Third, split counties and county segments, defined as any part of the county that is in a district not wholly within that county, shall each be as few as possible. Fourth, as few municipal lines shall be crossed as possible.”

  Although this requirement mentions municipal lines, and the MO-LWV expressed a desire to follow school district boundaries as well, we focus on maintaining county lines; too many very specific constraints can cause an automated algorithm to struggle to find good district plans. Therefore, the plans presented in Section 5 might not follow municipal lines or school district boundaries. For a similar reason, the algorithm does not explicitly enforce the second requirement (which refers to the remaining population of a county that wholly contains one or more districts). Therefore, the state legislative plans in this report may require manual adjustment to more effectively preserve political subdivisions.

  The algorithm uses a few different methods to preserve county lines. First, the plans in this report are constructed using a combination of counties, census tracts, census block groups, and census blocks. Although district plans are typically drawn using census blocks, the smallest geographic unit for which the census collects information, population balance is still achievable with these larger units. Automatically keeping counties with low population intact helps preserve county lines, and using other larger units also helps maintain compact districts.
In addition to using whole counties as geographic units to preserve county lines (when possible), we also rely on numeric “penalties” for districts that cross county lines. These penalties compel the algorithm to view a district that crosses a county line as less compact than a district wholly contained within a county, even if the two districts have the same perimeter. More details on these numeric penalties are included in Appendix A. For the senate plans specifically, we impose an additional constraint that requires counties to wholly contain as many districts as their population allows.

While the Missouri Constitution does not have explicit requirements concerning natural boundaries, there are some sections of the Missouri River that the current senate and house districts do not cross. The MO-LWV was divided on whether districts should be able to cross these sections of the river; to err on the side of the status quo, the senate plans in this report are designed to have districts that do not directly cross some sections of the river as well. Figure 7 show the specific sections of the river used for this restriction in the senate plans. Although districts cannot directly cross these sections of the river, they may wrap around them; for example, a district can only have area inside both St. Charles and St. Louis Counties if it also has area outside of those two counties, to serve as a connector.

Figure 7  Sections of the Missouri River that districts in the senate plan cannot directly cross (shown in black). To the west, there is a section between Jackson and Clay Counties; to the east, there is a section between St. Charles and St. Louis Counties.
• Partisan fairness:

Relevant text from the Missouri Constitution: “Districts shall be drawn in a manner that achieves both partisan fairness and, secondarily, competitiveness, but [population balance, no racial discrimination, contiguity, compactness, and the preservation of political subdivisions] shall take precedence over partisan fairness and competitiveness [...]

“To this end, the average electoral performance of the two political parties receiving the most votes in the three preceding general elections for governor, for United States Senate, and for President of the United States shall be calculated. This index shall be defined as the total votes received by each party in the three preceding general elections for governor, for United States Senate, and for President of the United States, divided by the total votes cast for both parties in these elections. Using this index, the total number of wasted votes for each party, summing across all of the districts in the plan shall be calculated [...] In any redistricting plan and map of the proposed districts, the difference between the two parties’ total wasted votes, divided by the total votes cast for the two parties, shall not exceed fifteen percent.

“To promote competitiveness, the electoral performance index shall be used to simulate elections in which the hypothetical statewide vote shifts by one percent, two percent, three percent, four percent, and five percent in favor of each party. The vote in each individual district shall be assumed to shift by the same amount as the statewide vote. In each of these simulated elections, the difference between the two parties’ total wasted votes, divided by the total votes cast for the two parties, shall not exceed fifteen percent.”

This requirement states that the plan’s Efficiency Gap and Shifted Efficiency Gap should be below 15%. All state legislative plans in this report have an Efficiency Gap/Shifted Efficiency Gap value below 15%. As Section 4.2 mentions, we use precinct-level voting from the required 2016, 2018, and 2020 elections to calculate these values (Voting and Election Science Team 2021).

4.3.2. Congressional Redistricting Requirements The U.S. Constitution requires congressional districts to be equi-populous and comply with the Voting Rights Act of 1965 (National Conference of State Legislatures 2021); the Missouri Constitution additionally
states that congressional districts “shall be composed of contiguous territory as compact and as nearly equal in population as may be” (Mo. Const. art. III, §45). Below we list how the plans in this report satisfy these requirements.

- **Population balance:** Typically, congressional district populations deviate from the ideal district population (i.e., the total state population divided by the number of districts) by at most one person (National Conference of State Legislatures 2021). Single-person population balance is achievable when districts are constructed with census blocks, the smallest geographic census unit. For computational efficiency, the congressional plans in this report are constructed with counties and census tracts (a larger census unit); hence, we allow districts to deviate by at most 0.5% from the ideal population. The plans presented can then be manually tuned to single-person population balance using census blocks without significantly altering district characteristics (e.g., compactness, partisan fairness).

- **Voting Rights Act:** Currently, Missouri’s Congressional District 1 is a majority-minority district with a non-Hispanic white minority and a Black/African American plurality (Ballotpedia 2015). Each of the congressional plans in this report has a similar majority-minority district.

- **Contiguity and compactness:** All congressional plans in this report are contiguous; some plans are explicitly optimized for compactness, while others are optimized for different fairness metrics. In the latter cases, compactness is maintained as much as possible.

5. **District Plans**

Here we present a collection of plans for the state senate, state house, and congressional districts in Missouri, following the constitutional requirements discussed in Section 4.3 and optimized for the different fairness metrics discussed in Section 2.

The Missouri Senate consists of 34 districts and the ideal district population is 181,027 people. Following the Missouri Constitution, district populations deviate from the ideal population by at most 3%, which allows a difference of at most 10,860 people between the most populated and least populated senate districts. Only eight of the 115 counties have large enough populations to contain whole districts (St. Louis, Jackson, St. Charles, St.
Louis City, Greene, Clay, Jefferson, and Boone). Since Boone County has enough population for exactly one district, it is kept intact; the other seven counties are split into census tracts. Franklin County is also split into census tracts, even though it cannot contain a whole district; since Franklin County is near the densely populated St. Louis area, splitting it into tracts gives the algorithm more flexibility to find plans that satisfy population balance. Hence, we construct the senate plans with a combination of 107 counties and 862 census tracts.

The Missouri House consists of 163 districts and the ideal district population is 37,760 people. Following the Missouri Constitution, district populations deviate from the ideal population by at most 3%, which allows a difference of at most 2,264 people between the most populated and least populated house districts. The plans are constructed with a combination of census block groups and census blocks; the 20 most populated block groups in Jackson, Clay, and Jefferson Counties combined are split into 733 blocks, and the remaining 5011 block groups are kept intact. Splitting counties into block groups at the beginning gives the algorithm more flexibility to find good plans; note that the final plans have many counties intact, since the algorithm is designed to preserve county lines as it optimizes compactness. Since it is more difficult to visually examine how well the house plans preserve county lines, Section 6.2 reports the final number of whole counties and the total number of counties spanned by the districts (for example, a district that lies in three different counties would add three to this total).

Missouri has eight congressional districts and the ideal district population is 769,364 people. As mentioned in Section 4.3.2, district populations deviate from the ideal population by at most 0.5%, which allows a difference of at most 7,692 people between the most populated and least populated congressional districts. The congressional plans are constructed with a combination of 79 counties and 836 census tracts; the 30 least populated counties, counties in the Kansas City area (Jackson and Clay Counties), and counties in the St. Louis area (St. Louis City and Jefferson, St. Charles, and St Louis Counties) are split into census tracts. Similar to the state house plans, splitting more counties into tracts at the beginning gives the algorithm more flexibility to find good plans; note that the final plans have more than 79 counties intact, since optimizing for compactness tends to preserve county lines as well.

The district plans in this section can be categorized in the following manner. For each type of plan, we provide two different options.
• **Compact plans:** These plans were optimized solely for compactness. No effort was made to improve other fairness metrics beyond the 15% Efficiency Gap/Shifted Efficiency Gap requirements for state legislative plans.

• **Efficiency Gap plans:** First, these plans were optimized with respect to the Efficiency Gap; i.e., the algorithm tries to make this value as small as possible. Once the plans have a small Efficiency Gap value, the algorithm maintains this small value while improving compactness.

• **Partisan Asymmetry plans:** Similar to the approach taken to produce the Efficiency Gap plans, the algorithm first decreases the Partisan Asymmetry value as much as possible, and then improves compactness while maintaining a small value.

• **Competitive plans:** For these plans, the algorithm simultaneously improves compactness and competitiveness; every competitive district encountered while improving compactness is maintained.

Note that we construct some sets of district plans using multiple algorithm phases (i.e., first a plan is optimized for a partisan fairness metric, then the plan is optimized for compactness). Appendix A gives additional details for plan construction, including a discussion of these phases.

**For the state senate/house plans, we only present compact plans.** Although we ran experiments to optimize senate/house plans for the Shifted Efficiency Gap, Partisan Asymmetry, and competitiveness, we do not include them in this report. In these experiments, the algorithm was unable to improve these fairness metrics while following constitutional requirements (e.g., the preservation of political subdivisions).

### 5.1. Senate Plans - Compactness

Figures 8 and 9 display the compact district plans for the senate (Senate-COMP1 and Senate-COMP2, respectively). The districts in these plans are mostly comprised of whole counties, and each plan splits eight counties. For both plans, St. Louis City, Jefferson County, and Franklin County are each split between two districts, Clay County is split between three districts, and St. Louis County is split between seven districts. In Senate-COMP1, St. Charles County and Greene County are split between four districts; in Senate-COMP2, St. Charles County and Greene County are split between three districts. For both plans, Jackson County completely contains four districts, Clay County contains one, Greene County contains one, St. Charles County contains two, Jefferson County contains
one, St. Louis County contains five, and St. Louis City contains one. As mentioned at the beginning of this section, Boone County constitutes an entire district since it has sufficient population for exactly one district.

Although these plans were optimized for compactness, Senate-COMP1 includes long districts in Jackson County (Figure 8b), and Senate-COMP2 includes a long district in the center (Figure 9a). These slightly elongated shapes may be the result of satisfying population balance while simultaneously maintaining a large number of whole counties, and/or satisfying the 15% Efficiency Gap/Shifted Efficiency Gap threshold. It may be possible to further improve compactness by splitting more counties.

5.2. **House Plans - Compactness**

Figures 10 and 11 display the compact district plans for the house (House-COMP1 and House-COMP2, respectively). Both plans keep more than half of the counties intact. House-COMP1 splits fewer counties and its districts span fewer counties than House-COMP2. However, House-COMP1 has a few districts that consist of two areas joined by a thin connector (Figures 10b, 10c, and 10d). House-COMP2 has more whole districts contained within St. Louis City, and Clay, Greene, Jefferson, and St. Charles Counties than House-COMP1.

5.3. **Congressional Plans - Compactness**

Figures 12 and 13 display the compact congressional district plans (Congress-COMP1 and Congress-COMP2, respectively). Both plans keep the majority of counties intact; Congress-COMP1 has five split counties, while Congress-COMP2 has six. As mentioned in Section 4.3.2, these plans allow district populations to deviate from the ideal population by at most 0.5%; therefore, tuning these plans to single-person population balance may require additional split counties. In Congress-COMP2, the majority of Kansas City is contained within one district (Figure 13b), while Congress-COMP1 splits Kansas City between two districts (Figure 12b). Both plans split the St. Louis are between two districts (Figure 12c and 13c).

5.4. **Congressional Plans - Efficiency Gap**

Figures 14 and 15 display the Efficiency Gap congressional district plans (Congress-EG1 and Congress-EG2, respectively). As with the compact plans, most counties are kept intact; Congress-EG1 splits five counties and Congress-EG2 splits six. Both plans divide Kansas
City evenly between two districts (Figures 14b and 15b). While the compact plans have one long district that contains all of Missouri’s northern counties, Congress-EG1/2 split this region between two districts (Figures 14a and 15a). In both plans, the northern district in the St. Louis area (the majority-minority district) reaches further west than in the compact plans, while the southern district has more area in St. Louis City (Figures 14c and 15c). Although Congress-EG1/2 are not as compact as Congress-COMP1/2, the district shapes remain reasonable.

5.5. **Congressional Plans - Partisan Asymmetry**

Figures 16 and 17 display the Partisan Asymmetry congressional district plans (Congress-PA1 and Congress-PA2, respectively). As with the previous two sets of plans, most counties are kept intact; both plans split six counties. Compared to Congress-COMP1/2 and Congress-EG1/2, the southern district in the St. Louis area is more contained in the east (Figures 16c and 17c). In both plans, the district containing northern Kansas City is more localized than in Congress-COMP1 or Congress-EG1/2 (Figures 16b and 17b). For Congress-PA2, the southwest district containing Springfield reaches further north than in Congress-COMP1/2 (Figure 17a). These differences in district boundaries may result from the algorithm trying to improve Partisan Asymmetry by more thoroughly redistributing Democratic voters.

5.6. **Congressional Plans - Competitiveness**

Figures 18 and 19 display the competitive congressional district plans (Congress-CMPTTV1 and Congress-CMPTTV2, respectively). Similar to Congress-COMP1, both plans split five counties. The most notable difference between CMPTTV1/2 and the previous plans is that Jackson County is evenly split between three districts (Figures 18b and 19b). In Congress-CMPTTV2, the majority-minority district is located further south than in the previous plans (Figure 19c).
Figure 8 A senate plan optimized for compactness (Senate-COMP1), with a closer view of populated areas. District boundaries are shown in black and county boundaries that do not coincide with a district boundary are shown in white.
Figure 9  A senate plan optimized for compactness (Senate-COMP2), with a closer view of populated areas. District boundaries are shown in black and county boundaries that do not coincide with a district boundary are shown in white.
Figure 10 A house plan optimized for compactness (House-COMP1), with a closer view of populated areas. District boundaries are shown in black and county boundaries that do not coincide with a district boundary are shown in white.
(a) House-COMP2

(b) Jackson and Clay Counties (the Kansas City area)
(c) St. Charles, Jefferson, St. Louis City, and St. Louis Counties (the St. Louis area)
(d) Greene County (Springfield)

Figure 11  A house plan optimized for compactness (House-COMP2), with a closer view of populated areas. District boundaries are shown in black and county boundaries that do not coincide with a district boundary are shown in white.
Figure 12  A congressional plan optimized for compactness (Congress-COMP1), with a closer view of populated areas. District boundaries are shown in black and county boundaries that do not coincide with a district boundary are shown in white. The majority-minority district is hatched with black stripes.
Figure 13 A congressional plan optimized for compactness (Congress-COMP2), with a closer view of populated areas. District boundaries are shown in black and county boundaries that do not coincide with a district boundary are shown in white. The majority-minority district is hatched with black stripes.
Figure 14  A congressional plan optimized for the Efficiency Gap (Congress-EG1), with a closer view of populated areas. District boundaries are shown in black and county boundaries that do not coincide with a district boundary are shown in white. The majority-minority district is hatched with black stripes.
Figure 15 A congressional plan optimized for the Efficiency Gap (Congress-EG2), with a closer view of populated areas. District boundaries are shown in black and county boundaries that do not coincide with a district boundary are shown in white. The majority-minority district is hatched with black stripes.
Figure 16 A congressional plan optimized for Partisan Asymmetry (Congress-PA1), with a closer view of populated areas. District boundaries are shown in black and county boundaries that do not coincide with a district boundary are shown in white. The majority-minority district is hatched with black stripes.
Figure 17 A congressional plan optimized for Partisan Asymmetry (Congress-PA2), with a closer view of populated areas. District boundaries are shown in black and county boundaries that do not coincide with a district boundary are shown in white. The majority-minority district is hatched with black stripes.
Figure 18  A congressional plan optimized for competitiveness (Congress-CMPTTV1), with a closer view of populated areas. District boundaries are shown in black and county boundaries that do not coincide with a district boundary are shown in white. The majority-minority district is hatched with black stripes.
Figure 19  A congressional plan optimized for competitiveness (Congress-CMPTTV2), with a closer view of populated areas. District boundaries are shown in black and county boundaries that do not coincide with a district boundary are shown in white. The majority-minority district is hatched with black stripes.
6. Discussion

This section reports the fairness metric values for each plan, examines the trade-offs between plans, and discusses the fairness metric values that are achievable for Missouri given its voter distribution and redistricting requirements.

6.1. Senate Plans

The Senate-COMP1/2 plans in Section 5.1 prioritize compactness, in addition to maintaining county lines; therefore, these plans effectively follow Missouri’s constitutional requirements. However, it is also important to examine how prioritizing compactness and preserving county lines affects the partisan characteristics of these district plans. Table 1 shows that the Efficiency Gap/Shifted Efficiency Gap values for both plans are below 15%, but by small margins. Figures 20a and 20b show that Democrats lose many districts by small margins, causing them to waste many votes; for example, Democrats are expected to lose six of the ten districts that are within a 10% margin of victory. The Partisan Asymmetry values in Table 1 show that both plans have average symmetry; the vote-shares in Figures 20a and 20b do not show extreme skews (such as the skew depicted in Figure 2d). These values indicate that neither party has an extreme advantage as the state-wide voter preference shifts in either party’s favor.

<table>
<thead>
<tr>
<th></th>
<th>Comp</th>
<th>EG</th>
<th>SEG</th>
<th>PA</th>
<th>Cmpttv</th>
<th>D/R</th>
<th>MM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Senate-COMP1</td>
<td>15,240</td>
<td>12.89%</td>
<td>14.97%</td>
<td>0.039</td>
<td>10</td>
<td>10/24</td>
<td>3</td>
</tr>
<tr>
<td>Senate-COMP2</td>
<td>15,574</td>
<td>13.07%</td>
<td>14.99%</td>
<td>0.040</td>
<td>10</td>
<td>10/24</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 1: The metric values for the senate plans. From left to right, the table lists each plan’s compactness (Comp), Efficiency Gap (EG), Shifted Efficiency Gap (SEG), Partisan Asymmetry (PA), number of seats within a 10% margin of victory (Cmpttv), number of Democratic and Republican seats (D/R), and number of majority-minority districts (MM).

It is certainly possible that Missouri’s unique political geography causes district plans to have pro-Republican biases, even when districting goals are not explicitly political; most Democratic voters are concentrated in urban centers, so optimizing for compactness can pack urban districts with Democrats. We explored this possibility using additional plans that prioritize partisan fairness metrics (e.g., the Shifted Efficiency Gap). However, as mentioned in Section 5, the algorithm was unable to improve these fairness metrics while following constitutional requirements. For example, the algorithm was only able to improve
the Shifted Efficiency Gap by approximately 1% while maintaining the same number of whole counties and districts wholly contained within counties; this minor improvement also caused a few irregular district shapes. Similarly, the algorithm was only able to improve Partisan Asymmetry by approximately 0.005; this minor improvement caused district plans to violate the 15% Shifted Efficiency Gap requirement.

From these results, we conclude that there are limited options for senate plans that strictly adhere to Missouri’s constitutional requirements. Compactness, the preservation of political subdivisions, and the 15% Shifted Efficiency Gap threshold severely restrict the definition of an acceptable plan; enforcing these requirements prevents significant improvement to partisan fairness metrics.

6.2. House Plans

Similar to the senate plans, Missouri’s political geography produces an inherent Republican advantage, since Democrats are concentrated in a few urban centers. For the house plans specifically, the districts are too small to be flexible; they cannot stretch between urban and rural areas to achieve a Democrat-Republican balance. Table 2 shows that both plans have high Efficiency Gap/Shifted Efficiency Gap values; we can see from Figures 21a and 21b that Democrats are expected to waste votes from winning districts by large margins and losing districts by narrow margins. House-COMP1/2 also have average Partisan Asymmetry values; the vote-shares in Figures 21a and 21b are not extremely skewed.
Table 2  The metric values for the house plans. From left to right, the table lists each plan’s compactness (Comp), Efficiency Gap (EG), Shifted Efficiency Gap (SEG), Partisan Asymmetry (PA), number of seats within a 10% margin of victory (Cmpttv), number of Democratic and Republican seats (D/R), number of whole counties (Whole), number of counties spanned by the districts (Spanned), and number of majority-minority districts (MM).

<table>
<thead>
<tr>
<th>Plan</th>
<th>Comp</th>
<th>EG</th>
<th>SEG</th>
<th>PA</th>
<th>Cmpttv</th>
<th>D/R</th>
<th>Whole</th>
<th>Spanned</th>
<th>MM</th>
</tr>
</thead>
<tbody>
<tr>
<td>House-COMP1</td>
<td>55,759</td>
<td>14.83%</td>
<td>14.97%</td>
<td>0.038</td>
<td>51</td>
<td>46/117</td>
<td>63</td>
<td>293</td>
<td>21</td>
</tr>
<tr>
<td>House-COMP2</td>
<td>57,449</td>
<td>14.89%</td>
<td>14.94%</td>
<td>0.038</td>
<td>47</td>
<td>46/117</td>
<td>61</td>
<td>295</td>
<td>19</td>
</tr>
</tbody>
</table>

Figure 21  The estimated fraction of votes won by Democrats/Republicans in each district for the house plans (based on past election results). Democratic fractions are shown on the bottom in blue and Republican fractions are shown on the top in red.

As with the senate plans, we attempted to create house plans that prioritized partisan fairness metrics; however, the algorithm was unable to improve these metrics while following constitutional requirements. For example, the algorithm was only able to improve the Shifted Efficiency Gap by approximately 1.5%; this minor improvement caused a few irregular district shapes and caused populated counties to contain fewer whole districts. Similarly, further improvement to Partisan Asymmetry or the number of competitive districts caused district plans to violate the 15% Shifted Efficiency Gap requirement.

The house plans reiterate the conclusion of the senate plans: there are limited options for house plans that satisfy Missouri’s constitutional requirements of compactness, the preservation of political subdivisions, and the 15% Shifted Efficiency Gap threshold.

6.3. Congressional Plans

The Congress-COMP1/2 plans in Section 5.3 prioritize compactness, and hence they most effectively satisfy Missouri’s constitutional requirements (out of the four sets of congres-
sional plans in this report). However, as mentioned in Section 6.1, optimizing solely for compactness can unintentionally produce a Republican advantage. Table 3 shows that Congress-COMP1/2 have high Efficiency Gap values (17.83% and 18.50%, respectively). Figures 22a and 22b show that Democrats are expected to lose several districts by small margins, causing them to waste many votes. In contrast, Republicans win multiple districts by small margins, causing them to waste fewer votes. The Partisan Asymmetry values in Table 3 show that both plans have average symmetry; as with the senate/house plans, the vote-shares in Figures 22a and 22b are not extremely asymmetric.

<table>
<thead>
<tr>
<th>Plan</th>
<th>Comp</th>
<th>EG</th>
<th>PA</th>
<th>Cmpttv</th>
<th>D/R</th>
<th>MM-B</th>
<th>MM-W</th>
</tr>
</thead>
<tbody>
<tr>
<td>Congress-COMP1</td>
<td>4,219</td>
<td>17.83%</td>
<td>0.032</td>
<td>3</td>
<td>2/6</td>
<td>44.42%</td>
<td>42.53%</td>
</tr>
<tr>
<td>Congress-COMP2</td>
<td>4,398</td>
<td>18.50%</td>
<td>0.038</td>
<td>2</td>
<td>2/6</td>
<td>45.97%</td>
<td>40.67%</td>
</tr>
<tr>
<td>Congress-EG1</td>
<td>4,506</td>
<td>2.47%</td>
<td>0.015</td>
<td>3</td>
<td>3/5</td>
<td>37.27%</td>
<td>49.19%</td>
</tr>
<tr>
<td>Congress-EG2</td>
<td>4,991</td>
<td>2.48%</td>
<td>0.013</td>
<td>3</td>
<td>3/5</td>
<td>39.35%</td>
<td>47.48%</td>
</tr>
<tr>
<td>Congress-PA1</td>
<td>4,478</td>
<td>3.96%</td>
<td>0.010</td>
<td>3</td>
<td>3/5</td>
<td>39.73%</td>
<td>47.48%</td>
</tr>
<tr>
<td>Congress-PA2</td>
<td>4,740</td>
<td>15.46%</td>
<td>0.009</td>
<td>4</td>
<td>2/6</td>
<td>38.33%</td>
<td>48.76%</td>
</tr>
<tr>
<td>Congress-CMPTTV1</td>
<td>4,725</td>
<td>30.38%</td>
<td>0.033</td>
<td>4</td>
<td>1/7</td>
<td>46.46%</td>
<td>40.39%</td>
</tr>
<tr>
<td>Congress-CMPTTV2</td>
<td>5,349</td>
<td>18.49%</td>
<td>0.029</td>
<td>4</td>
<td>2/6</td>
<td>39.5%</td>
<td>47.40%</td>
</tr>
</tbody>
</table>

Table 3  The metric values for the congressional plans. From left to right, the table lists each plan’s compactness (Comp), Efficiency Gap (EG), Partisan Asymmetry (PA), number of seats within a 10% margin of victory (Cmpttv), number of Democratic and Republican seats (D/R), the percentage of the majority-minority district that is Black/African American (MM-B), and the percentage of the majority-minority district that is non-Hispanic white (MM-W).

We also consider congressional plans that prioritize compactness and the Efficiency Gap, Partisan Asymmetry, or competitiveness. Since there are fewer districts and fewer requirements for congressional redistricting in Missouri than for state legislative redistricting, the algorithm is able to significantly improve these metrics while maintaining population balance, compactness, and a majority-minority district.

For the Efficiency Gap plans, Table 3 shows that Congress-EG1/2 have good Efficiency Gap values below 2.5% (compared with values above 17% for Congress-COMP1/2). These low values indicate that both parties waste a similar percentage of votes. Table 3 and Figures 22c and 22d show that Democrats are expected to narrowly win a third seat that
Figure 22 The estimated fraction of votes won by Democrats/Republicans in each district for the congressional plans (based on past election results). Democratic fractions are shown on the bottom in blue and Republican fractions are shown on the top in red.
they narrowly lose in Congress-COMP1/2, causing them to waste fewer votes. Significantly improving the Efficiency Gap also improves Partisan Asymmetry, since the district votes shares in Figures 22c and 22d are distributed slightly more symmetrically. Congress-EG1/2 both have three competitive districts; Democrats are expected to narrowly win two of these competitive districts and Republicans are expected to win one (Figures 22c and 22d).

For the Partisan Asymmetry plans, Table 3 shows that Congress-PA1/2 have excellent Partisan Asymmetry values less than or equal to 0.01 (compared with values around 0.035 for Congress-COMP1/2). These low values indicate that district vote-shares are quite symmetric; Figures 22e and Figures 22f show that Democratic voters are spread more evenly throughout the districts. Congress-PA1/2 also have 3-4 competitive districts. For Congress-PA1, Democrats are expected to win three districts while Republicans are expected to win five; for Congress-PA2, Democrats are expected to win two districts while Republicans are expected to win six. This difference in expected seat outcomes explains the difference in Efficiency Gap values (3.96% for Congress-PA1 and 15.46% for Congress-PA2). Democrats waste more votes in Congress-PA2 by narrowly losing a competitive district that they narrowly win in Congress-PA1.

Lastly, both of the competitive plans have four competitive districts (shown in Table 3). Congress-CMPTTV1/2 highlight the relationship between competitiveness and the other partisan fairness metrics. In Congress-CMPTTV1, Republicans are expected to win all four of the competitive districts. Figure 22g shows that Democrats are cracked, since they are expected to lose all four competitive districts. Since Democrats waste many votes with this arrangement of vote-shares, Congress-CMPTTV1 has a large Efficiency Gap of 30.38%. In Congress-CMPTTV2, Republicans are expected to win three competitive districts and Democrats are expected to win one. This adjustment in district vote-shares decreases the Efficiency Gap to 18.49% (comparable to Congress-COMP2). Hence, competitive districts can exist without good values for the Efficiency Gap.

It is important to note that improving partisan fairness metrics for the congressional plans tends to decrease the majority-minority district’s Black/African American population. As mentioned in Section 4.3.2, the majority-minority district in Missouri’s current congressional district plan has a Black/African American plurality; however, this is not always the case for the congressional plans in this report. Table 3 shows that the majority-minority district’s population in Congress-COMP1/2 and Congress-CMPTTV1 is roughly
45% Black/African American, compared to roughly 39% in Congress-EG1/2, Congress-PA1/2, and Congress-CMPTTV2. While the algorithm sought to improve the Efficiency Gap, Partisan Asymmetry, or competitiveness by redistributing Democratic voters more evenly throughout the districts, it also redistributed the Black/African American population in the St. Louis area. Although the white population still constitutes a minority in this district for Congress-EG1/2, Congress-PA1/2, and Congress-CMPTTV2, it also constitutes a plurality.

Missouri’s unique political geography causes state legislative and congressional district plans optimized for compactness to have pro-Republican biases. However, unlike in the state legislative plans, partisan fairness can be substantially improved in the congressional plans (without sacrificing compactness). A congressional plan with an Efficiency Gap value around 2.5%, a Partisan Asymmetry value around 0.01, and/or 3-4 competitive districts is certainly achievable for Missouri; a district plan with these characteristics could simultaneously satisfy compactness and partisan fairness.

6.4. Limitations
The district plans constructed for this report provide insight into how Missouri’s constitutional requirements and political geography affect the redistricting process. However, there are limitations to consider that impact the optimization algorithm’s ability to fully consider all possible district plans and constitutional requirements in Missouri. Hence, this report should be used as a reference to understand fairness metrics and the affects of political geography and redistricting requirements on partisan fairness, rather than an exact prescription for drawing Missouri districts.

First, we construct the congressional district plans with counties and census tracts, rather than census blocks. Although we enforce a strict population balance requirement, single-person population balance is not achieved. Therefore, the congressional district plans in this report would require manual adjustment to achieve single-person population balance. We also construct the state legislative district plans with a combination of counties, census tracts, census block groups, and census blocks, rather than solely with census blocks. Although the constitutionally required population balance is achievable with these larger geographic units, using blocks could provide more district plan options.

There are also limitations regarding the redistricting requirements that the algorithm enforces. As Section 4.3 discusses, we do not explicitly incorporate racial data into the
algorithm for state legislative plans; hence, there is a chance that the state legislative plans in this report might unintentionally weaken the voting power of a particular minority. To provide a starting point for further discussion on this constitutional requirement, we include the number of majority-minority districts in each senate and house plan (Table 1 in Section 6.1 and Table 2 in Section 6.2, respectively). Section 4.3 also mentions that we do not attempt to preserve municipal lines or school district boundaries in the state legislative plans. Similarly, we do not explicitly enforce the requirement that the remaining population of a county that contains one or more districts should be wholly joined in a single district. These limitations mean that the state legislative district plans in this report might require manual adjustment to improve the representation of racial/ethnic minorities and adequately preserve political subdivisions.
Appendix A: Additional Technical Details

In this appendix we discuss some of the technical details for creating district plans with the optimization algorithm. As mentioned in Section 3, the algorithm we use is based on a method from DeFord et al. (2019). Their paper fully explains the mechanics behind how to alter the boundary between two districts in each step of the algorithm. While their goal is to create a large sample of legal district plans for statistical analysis, our goal is to iteratively find better and better plans (with respect to some fairness metric) until there is no more room for significant improvement.

The algorithm steps from Section 3 are applied in multiple phases; these phases are distinguished by the initial plan the algorithm uses, the metric the algorithm improves, and the redistricting constraints the algorithm enforces. Note that the basic steps of the algorithm from Section 3 do not change between phases. Figures 23 and 24 outline these phases and the rest of this appendix provides additional details.

For the senate, house, and congressional plans, we begin with an approximation of the plan currently in place. Since the current plans are drawn at the census block level, some census tract/block groups are split. We assign split tracts/block groups to the district with which they share the most area. Next, we optimize these plans for population balance (this is Phase 1 in Figures 23 and 24); the algorithm transitions between contiguous plans with smaller and smaller population deviations until the desired population balance is satisfied. For the state senate plans, the algorithm requires counties to wholly contain as many districts as their population allows at each iteration; this constraint is enforced for all of the other algorithm phases described in the following paragraphs. Similarly, the algorithm requires a majority-minority district in each phase for the congressional plans. The algorithm alternates between two methods to choose which two districts to modify during an iteration. For the first method, the algorithm first chooses the district with the largest population deviation, then chooses a random neighboring district. For the second method, the algorithm first chooses a district at random, then chooses a random neighboring district. We alternate between these methods to prevent the algorithm from getting stuck in a local minimum; improving population balance throughout the state creates more opportunities to shift population to/from the district with the largest population deviation. Achieving population balance takes roughly 1,000-3,500 iterations for the senate (about 15 minutes), 4,000-5,000 iterations for the house (about 30 minutes), and 40 iterations for congressional (about 5 minutes). Note that the computation times for the senate, house, and congressional plans differ because they have different numbers of districts and are constructed with different numbers of geographic units. We collect 2-3 senate/house/congressional plans that satisfy population balance to serve as initial plans for future algorithm phases. Section 5 then presents the two best plans for each metric.

For the senate and house plans, we additionally improve the Shifted Efficiency Gap values of the population-balanced plans until they satisfy the constitutionally required 15% threshold (this is Phase 2 in Figure 23). The algorithm begins with the population-balanced plans, then transitions between contiguous, population-balanced plans that gradually improve the Shifted Efficiency Gap. For these plans, and for the remaining experiments, the algorithm chooses which two districts to modify at each iteration in the following manner. First, the districts are listed in a random order. At each iteration, the algorithm chooses the next district in this list, then chooses a random neighboring district. When the algorithm has completed enough iterations
Figure 23  A flowchart with the basic optimization algorithm phases used to construct state legislative district plans. The algorithm first improves population balance and the Shifted Efficiency Gap to satisfy constitutional requirements. Then the algorithm can optimize compactness or a partisan fairness metric. For Missouri state senate plans, we impose an additional constraint in each phase that requires counties to wholly contain as many districts as their population allows.

To finish the list (i.e., has completed a cycle), a new list of districts is created, in a new random order. Cycling through the districts ensures that all areas of the state are changing at the same pace. Satisfying the required 15% threshold takes roughly 10-20 cycles for the senate (about 10 minutes) and roughly 30-40 cycles for the house (about 2 hours).

Next, we optimize the senate, house, and congressional plans solely for compactness (this is Phase 3 in Figure 23 and Phase 2 in Figure 24). The algorithm begins with the population-balanced plans, then
Figure 24 A flowchart with the basic optimization algorithm phases used to construct congressional district plans. The algorithm first optimizes population balance, then compactness, and then can proceed to optimize a partisan fairness metric.

Transitions between contiguous, population-balanced plans with better and better compactness scores until the scores appear to plateau. Improving compactness takes roughly 50-150 cycles for the senate (about 30 minutes to 1 hour), roughly 130 cycles for the house (about 10 hours), and roughly 40-45 cycles for congressional (about 1-2.5 hours).

As mentioned in Section 4.3, to preserve county lines in state legislative plans we add numeric penalties to a plan’s compactness score when districts cross county lines. District perimeter segments that do not coincide with county lines receive a penalty; the length of these perimeter segments are multiplied by 4 when added to the sum of all district perimeters. District perimeter segments that do coincide with county lines receive no penalty; the true length of these perimeter segments are added to the sum of all district perimeters. Note that this penalty system does not penalize districts comprised of multiple whole counties. Other penalty multipliers were tested (such as 1.25, 1.5, 1.75, 2, 3, and 5), but these either produced little effect, or caused
misshapen districts; an extreme penalty such as 5 encouraged districts to cling to county lines, even if doing so created long tendrils.

As mentioned in Sections 5 and 6, we ran experiments to optimize state legislative plans for the Shifted Efficiency Gap, Partisan Asymmetry, and competitiveness, but the algorithm was unable to significantly improve these metrics. Optimizing for these partisan fairness metrics corresponds to Phases 3, 4, and 5 of Figure 23.

For the congressional plans, the algorithm was able to improve the Efficiency Gap, Partisan Asymmetry, and competitiveness. Optimizing the Efficiency Gap corresponds to Phase 3 in Figure 24. The algorithm begins with the compactness plans, then transitions between contiguous, population-balanced plans that satisfy a compactness threshold and gradually improve the Efficiency Gap. The compactness threshold is set roughly 2,000 miles above the initial plan’s perimeter sum (which is usually around 4,500 miles). The algorithm is run until the Efficiency Gap falls below a certain threshold; we use an Efficiency Gap threshold of 2.5%. Once the desired threshold is achieved, the algorithm optimizes the plans for compactness, while maintaining an Efficiency Gap value below the threshold (this is Phase 4 in Figure 24). The congressional plans take roughly 20 cycles (about 1 hour) to improve the Efficiency Gap and roughly 50 cycles (about 2 hours) to improve compactness.

We optimize congressional plans for Partisan Asymmetry in a manner described for the Efficiency Gap (using the appropriate Phase 3 for Partisan Asymmetry in Figure 24). The algorithm is run until the Partisan Asymmetry value falls below a threshold of 0.01. Once the desired threshold is achieved, the algorithm optimizes the plans for compactness, while maintaining a Partisan Asymmetry value below the threshold (this is Phase 4 in Figure 24). The congressional plans take roughly 30 cycles (about 1.5 hours) to improve Partisan Asymmetry and roughly 20 cycles (about 2-3 hours) to improve compactness.

Lastly, we optimize congressional plans for competitiveness (this is the appropriate Phase 2 in Figure 24). In contrast to the Efficiency Gap and Partisan Asymmetry plans, we improve competitiveness and compactness simultaneously. The algorithm begins with the population-balanced plans, then transitions between contiguous, population-balanced plans with better and better compactness scores. However, if the algorithm creates a competitive district during an iteration, that district is required to remain competitive for all future iterations. Therefore, as compactness improves, the number of competitive districts also increases. For the congressional plans, improving competitiveness and compactness simultaneously takes roughly 50 cycles (about 1.5-3 hours).
References


Mo Const art III, §3.

Mo Const art III, §45.


